CURRENT TRENDS IN THE SEISMIC ANALYSIS AND DESIGN OF HIGH RISE STRUCTURES

by

Nathan M. Newmark*

INTRODUCTION

The design of a structure is a process of synthesis, as contrasted with the analysis for given loadings or environmental conditions. In the design of a building to resist earthquake motions, the designer works within certain constraints such as: the architectural configuration of the building, the foundation conditions, the nature and extent of the hazard should failure or collapse occur, the possibility of an earthquake, the possible intensity of earthquakes in the region, the cost or available capital for construction, and similar factors. In the light of available information, the designer chooses the materials to be used, the method of construction, and the design concepts. He may choose to use steel, aluminum, concrete, masonry, or a combination of materials. He may select a frame with rigid connections, a frame with bracing, or a structure carrying lateral forces primarily by deep walls or "shear walls", or a combination of these elements.

But whatever the choice, the designer must have some basis for the selection of the strength and the proportions of the building and of the various members in it. The required strength depends on factors such as the intensity of earthquake motions to be expected, the flexibility of the structure, and its ductility or reserve strength before damage occurs. Because of the inter-relations among flexibility and strength of a structure, and the forces generated in it by earthquake motions, the dynamic design procedure must take these various factors into account. The ideal to be achieved is one involving appropriate flexibility and energy absorbing capacity, permitting the earthquake displacements to take place without unduly large forces being generated. Tc achieve this end, control of the construction procedures and appropriate inspection practices are necessary. The attainment of the ductility required to resist earthquake motions must be emphasized.

In the material presented here a general description of the response of relatively simple systems to earthquake motions is presented, first for elastic behavior, and then for inelastic behavior. Generalizations are made about the relation of the response of multi-degree-of-freedom systems to simple systems, both in the elastic and the inelastic range. Consideration it given to the general nature of the provisions

* Professor of Civil Engineering, University of Illinois, Urbana, Illinois.

of current building codes for earthquake resistant design. Based on the relations between the results of theoretical analyses and current design provisions, estimates are made of the required ductility for earthquake resistant design of buildings. Finally, some comments are made about the design of actual buildings and their behavior in earthquakes.

RESPONSE OF SIMPLE STRUCTURES TO EARTHQUAKE MOTIONS

A series of structures of varying size and complexity is shown in Fig. 1, corresponding to a simple, relatively compact machine anchored to a foundation in Item 1, a simple bent or frame in Item 2, a more complex frame in Item 3, multistory buildings of 15 stories in Item 4, and of 40 stories in Item 5, an elevated water tank in Item 6, and a suspension bridge responding either laterally or vertically in Item 7. The period of vibration, T, or the frequency of vibration, f, in the fundamental mode of vibration is indicated for each of these structures.

Each of the structures shown in Fig. 1 could be represented by a dashpot, as shown in Fig. 2. The relation between the circular frequency of vibration ω, the natural period f, and the period T, is given by the following equation in terms of the spring constant k, and the mass m:

$$\omega^2 = \frac{k}{m} \qquad (1)$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (2)$$

The effect of the dashpot is to produce damping of free vibrations or to reduce the amplitude of forced vibrarions, in general. The amount of damping is most conveniently considered in terms of the proportion of critical damping, β, which for most practical structures is relatively small, in the range of 0.5 to 10 or 20 per cent, and does not appreciably affect the natural period or frequency of vibration.

The simple system in Fig. 2 can be used to represent the various modes of vibration of a multi-degree-of-freedom system. For the time being, however, we shall consider only the fundamental mode of vibration of the multi-degree-of-freedom systems in Fig. 1, as represented by the single-degree system in Fig. 2.

When the base of the system, Fig. 2, moves with respect to time, the mass is set into motion also, and strains are induced in the spring. The motion of the base may be described by giving the displacement as a function of time, or, equally as descriptive of the motion, the time history of the velocity of the base, or the time history of its acceleration. Strong motion earthquake accelerations with respect to time have been obtained for a number of earthquakes. Ground motions from other sources of disturbances, such as quarry blasting, nuclear blasting, etc., are also available, and show many of the same characteristics. The most intense strong motion earthquake record that has been recorded so far is that of El Centro California earthquake of May 18, 1940. The recorded accelerogram for that earthquake, in the north south component of horizontal motion, is shown in Fig. 3. On the same figure are shown integration of the ground acceleration to give the variation of ground velocity with time, and the integration of velocity to give the variation of ground displacement with time. These integrations require base-line corrections of various sorts, and the magnitude of the maximum displacement may vary depending on how the corrections are made. The maximum velocity is relatively insensitive to the corrections, however. For this earthquake, with the integrations performed as shown in Fig. 3, the maximum ground acceleration is 0.32g, the maximum ground velocity 13.7 in/sec., and the maximum ground displacement is 8.3 in. These three maximum values are of particular interest because they help to define the response motions of the various structures considered in Fig. 1 most accurately, if all three maxima are taken into account.

For the ground motions in Fig. 3 or any other type of ground motion that might be considered, the response of the simple oscillator shown in Fig. 2 can be readily computed as a function of time. The maximum values of the response of this oscillator are of particular interest. These maximum values might be stated in terms of the maximum strain in the spring in Fig. 2, $u_m = D$; or alternatively, the response can be stated as the maximum spring force, or the maximum acceleration of the mass which is related to the maximum spring force directly when there is no damping; or by a quantity having the dimensions of velocity, which gives a measure of the maximum energy absorbed in the spring. This quantity, designated the pseudo-velocity is defined in such a way that the energy absorption in the spring is $\frac{1}{2}mV^2$. The relations among the maximum relative displacement of the spring, D, the pseudo-velocity, V, and the pseudo-acceleration, Ag, which is a measure of the force in the spring, are as follows:

$$V = \omega D \quad (3)$$

$$Ag = \omega V = \omega^2 D \quad (4)$$

The pseudo-velocity V is nearly equal to the maximum relative velocity for systems with moderate or high frequencies, but may differ considerably from the maximum relative velocity for very low frequency systems. The pseudo-acceleration A is exactly equal to the maximum acceleration for systems with no damping, and is not greatly different from the maximum acceleration for systems with moderate amounts of damping, over the whole range of frequencies from very low to very high values. Typical plots of the response of the system as a function of period or frequency are called response spectra. Response spectrum plots are shown in Fig. 4 for acceleration and for relative displacement, for a system with a moderate amount of damping, subjected to an input similar to that show in Fig. 3. This arithmetic plot of maximum response is simple and convenient to use. However, a somewhat more useful plot, which indicates at one and the same time the values for D, V, and A, is indicated in Fig. 5. This has the virtue that it also indicates more clearly the extreme or limiting values of the various parameters defining the response.

For Fig. 5, the frequency is plotted on a logarithmic scale. Since the frequency is the reciprocal of the period, the logarithmic scale for period would have exactly the same spacing of the points, or in effect the plot would be turned end for end.

The pseudo-velocity is plotted on a vertical scale, also logarithmically. Then on diagonal scales, as indicated on the figure, along an axis that extends upward from right to left are plotted values of the displacement, and along an axis that extends upward from left to right, the pseudo-acceleration, in such a way that any one point defines for a given frequency the displacement D, the pseudo-velocity V, or the pseudo-acceleration Ag. Points are indicated in Fig. 5 for the seven structures of Fig. 1, plotted at the fundamental frequencies for the structure considered. One can read directly from the plot the response values. Some further interpretation is needed for the response of a multi-degree-of freedom system, however. More detailed explanation of these points is given in Ref. 1.

The typical shape of the response spectrum shown in Fig. 5 is characteristic of the response for almost any type of input. A wide variety of motions have been considered in Refs. 2 and 3, ranging from simple pulses of displacement, velocity, or acceleration of the ground, through more complex motions such as those arising from nuclear blast detonations, and for a variety of earthquakes as taken from available strong motion records. Typical of all of these is the response spectrum shown in Fig. 6 of the same El Centro Earthquake, the motion records for which are given in Fig. 3. The response spectrum for small amounts of damping is much more jagged than indicated by Fig. 5, but for the higher amounts of damping the response curves are smooth. The scales are chosen in this instance to represent the amplifications of the response relative to the ground motion values of displacement, velocity, or acceleration.

The spectrum shown in Fig. 6 is typical of response spectra for nearly all types of ground motion. It is noted that on the extreme left of Fig. 6, corresponding to very low frequency systems, the response for all degrees of damping approaches an asymptote corresponding to the value of the maximum ground displacement. A low frequency system corresponds to one having a very heavy mass and a very light spring. When the ground is moved relatively rapidly, the mass does not have time to move, and therefore the maximum strain in the spring is precisely equal to the maximum displacement of the ground.

On the other hand, for a very high frequency system, the spring is relatively stiff and the mass very light. Therefore, when the ground is moved, the stiff spring forces the mass to move in the same way the ground moves, and the mass must therefore have the same acceleration as the ground at every instant. Hence, the force in the spring is that required to move the mass with the same acceleration as the ground, and the maximum acceleration of the mass is precisely equal to the maximum acceleration of the ground. This is shown by the fact that all of the lines on the extreme right-hand side of the figure approach as an asymtote the maximum ground acceleration line.

For intermediate frequency systems, there is an amplification of motion. In general the amplification factor for displacement is less than that for velocity, which in turn is less than that for acceleration. Amplification factors indicated by the figure are of the order of about 3.5 for displacement, 4.2 for velocity, and about 9.5 for acceleration, for the undamped system in Fig. 6. For damping of the order of about 10 per cent critical, these amplifications are slightly over 1 for displacement, about 1.5 for velocity, and about 2 for acceleration.

For an infinitely long harmonic oscillation, the amplification would become infinite for zero damping or would be limited by the amount of damping for systems with viscous damping. However, even in these cases the same general relationships are applicable.

The results of similar calculations for other ground motions are quite consistent shown with those in Fig. 6, even for simple motions. The general nature of the response spectrum is shown in Fig. 7, as consisting of a central region of amplified response, and two limiting regions of response, in which, for low frequency systems, the response displacement is equal to the maximum ground displacement, and for high frequency systems, the response acceleration is equal to the maximum ground acceleration. For damping of the order of about 5 to 10 per cent critical, the amplification factors for displacement, velocity, and acceleration, are only slightly over 1, 1.5, and 2.0, respectively, for a wide variety of earthquake and ground shock motions. These amplification factors increase quite rapidly, however, as the damping decreases, and decrease relatively slowly as the damping factor increases from the values of 5 to 10 per cent.

RESPONSE SPECTRA FOR INELASTIC SYSTEMS

A typical inelastic spring force-displacement relation is shown in Fig. 8. This can be approximated by an elasto-plastic relation as indicated, with an elastic initial region, a plastic ceiling of constant resistance, and an elastic unloading. The unloading is considered to be elastic until yielding is reached in the opposite direction. For equal yield values in either direction, calculations of the response of the system of Fig. 2 for an elasto-plastic resistance function can be made.

A variety of such calculations have been made and are reported in Refs. 2 and 4. It is interesting and instructive to plot the results of such calculations on a chart similar to the tripartite response spectrum charts of Figs. 5 and 6. This can be done directly for the elasto-plastic system, for constant values of ductility factor μ, which is the ratio of the maximum relative displacement of this spring to the yield point value of displacement. However, the plot can be made only in terms of the elastic component of displacement, in which case the accelerations are properly presented; or alternatively for the total displacement, in which case the accelerations are not properly presented. Since the former case is most convenient, this is what has been used as a basis for the chart shown in Fig. 9, which is presented also for the El Centro Earthquake, for elasto-plastic systems having varied amounts of yielding, but with a damping factor of 2 per cent of critical in the elastic range of the response. Ductility factors ranging from 1, or elastic behavior, up to 10, are shown in the figure. The total displacements can be obtained directly from the figure by multiplying the displacement components by the value of μ, the ductility factor, assigned to each curve. It is noted in Fig. 9 that the displacements vary roughly inversely as the ductility factor for the left-hand side of the chart or for low or even intermediate frequency systems, and the accelerations are nearly the same for all high frequency systems. This is consistent with the observations made earlier, for low frequency systems, that the spring displacement is equal to the maximum ground displacement, and this is true regardless of the nature of the spring or force displacement curve. For high frequency systems, the acceleration of the mass must be the same as the ground, and, therefore, the acceleration must be the same as the ground acceleration regardless of the characteristics of the spring.

The results obtained in Fig. 9, and from similar calculations for other earthquake and ground shock motions, are approximated in Fig. 10. In the left-hand side of this figure it is apparent that the total displacement is the same for the elasto-plastic response as for an elastic system; and for the right-hand side, the acceleration is the same for the elasto-plastic system as for the elastic system. In the intermediate region, one can approximate the results by use of the relationship that the energy is the same for the elasto-plastic system as for an elastic systemhaving the same frequency. These observations lead to further generalizations which have been verified by additional calculations some of which are reported in Ref. 2, but others are still under study. The generalizations may be stated as follows:

(1) For low frequency systems, the total displacement for the inelastic system is the same as for an elastic system having the same frequency.

(2) For intermediate frequency systems, the total energy absorbed by the spring is the same for the inelastic system as for an elastic system having the same frequency.

(3) For high frequency systems, the force in the spring is the same for the inelastic system as for an elastic system having the same frequency.

A number of stress-strain curves are shown in Fig. 11, and the above-mentioned rules are indicated in that figure. In the region where displacement is preserved, the forces or accelerations vary in accordance with the ordinates to the curve of stress or force relative to strain, at a constant strain ordinate. In the region where energy is preserved, both the displacement and the force vary in such a way as to keep the area under the various curves the same for all of the curves shown. In the region where force is preserved, the displacement varies in accordance with the displacement abscissas for a constant value of stress.

Shown in Fig. 11 are lines corresponding to (1) an elastic resistance, (2) an inelastic resistance having the same initial slope, (3) an inelastic resistance showing a maximum and a decay beyond the maximum, and (4) an elasto-plastic resistance, as indicated by the dashed line.

However, Fig. 11 indicates that the line for a force greater than the yield point force for the elasto-plastic curve will never intersect the elasto-plastic curve, and will, therefore, give an infinite displacement. This is not the case under actual conditions. Limits that are more realistic can be obtained if one plots the response spectrum in terms of total relative displacement for the inelastic curves, as indicated in Fig. 12. Here the lower set of lines corresponding to $\overline{D}$, $\overline{V}$, and $\overline{A}$ are for an elastic condition. The curves $\overline{D}_1$, $\overline{V}_1$, $\overline{A}_1$ are drawn for an inelastic force-displacement relation in which displacement is preserved for $\overline{D}_1$, energy is preserved for $\overline{V}_1$, and acceleration is preserved for $\overline{A}_1$. Similar curves are shown for $\overline{V}_2$ and $\overline{A}_2$ at somewhat higher levels of inelastic behavior. Curves for other levels can also be drawn, say for $\overline{V}_3$ and $\overline{A}_3$, for example.

At a frequency such as f_a, all of the displacements are bounded by point a, and the displaced limit is the same for all of the inelastic curves considered.

At a frequency such as f_b, the displacement limit corresponds to point B_0 on $\overline{V}$, and a greater displacement corresponding to point b_1 on $\overline{V}_1$. However, one cannot reach $\overline{V}_2$ without crossing the line corresponding to $\overline{D}$. Hence, the upper bound of displacement is given by $\overline{b}$.

At a frequency such as f_c, the displacement bound is given by c_0 on line $\overline{A}$, c_1 on line $\overline{A}_1$, but by c_2 on $\overline{V}_2$, which intersection is reached before the line reaches $\overline{A}_2$. Similarly the upper bound is $\overline{c}$ on line $\overline{D}$, which is below $\overline{V}_3$.

In other words, for inelastic systems, for low frequencies, displacement is preserved. For intermediate frequencies, energy is preserved, except that the displacement can not be greater than the displacement bound for the elastic response spectrum. Furthermore, for high frequency systems, force (or acceleration) is preserved, provided that the energy absorbed is not greater than the energy bound for the elastic spectrum, and provided also that the displacement is not greater than the displacement bound for the elastic response spectrum. It should be pointed out that the relative values to be used here for the elastic response spectrum should be those corresponding to about 5 to 10 percent of damping; otherwise, the acceleration and velocity bounds will be too high.

MULTI-DEGREE-OF-FREEDOM SYSTEMS

A multi-degree-of-freedom system has a number of different modes of vibration. For example, the shear beam shown in Fig. 13 has a fundamental mode of lateral oscillation as shown in Fig. 13(b), a second mode as shown in Fig. 13(c), and a third mode as shown in Fig. 13(d). Each of these modes can be considered to vibrate independently, with participation factors as defined in the usual way, and as described in detail in Ref. 1. A response spectrum for a multi-degree-of-freedom system can be drawn for a particular system, as a function of the fundamental frequency of the system, in much the same way as a response spectrum for a single-degree-of-freedom system is drawn. We may do this so as to define displacement bounds D', V', and A', for the multi-degree-of-freedom system, which are drawn in such a way that they can be used instead of D, V and A to give the response values desired for the multi-degree-of-freedom system, when the fundamental frequency is used to define the frequency for the response spectrum value for the multi-degree-of-freedom system. These curves then involve the participation factors and modal responses for the various modes. The relationships are shown schematically in Fig. 14. Further exploration of these concepts is under way and is presented in somewhat more detail in Refs. 5 and 6.

For horizontal motions of the base of a structure founded on a firm foundation without rocking, the participation factors of the various modes can be selected by proper choice of the modal values, as indicated in Ref. 1, so as to make participation factors unity. When the modal values are so chosen, a particular response parameter at a particular point in the structure, α has values for each of the modes designated by α (for the nth mode). The quantity α may be an acceleration of a particular mass, a strain at a particular point, a moment at a particular joint in a particular member, a shear in a particular story, displacement of a particular node or joint, etc. If for every frequency f_n of the structure there is defined by the response spectrum a relative displacement response value of D_n, then because the various modal maxima cannot occur simultaneously, an upper bound to the particular response quantity is given by the following relation:

$$\bar{\alpha} \leq \sum | \alpha_n D_n | \qquad (5)$$

This equation indicates that the actual response quantity $\bar{\alpha}$ is less than or equal to the sum of the absolute values of all of the modal response values, each of which is equal to the response value for that mode, α_n, multiplied by the spectrum displacement value for that mode, D_n, provided that the participation factor for the nth mode is unity.

Relations equivalent to Eq. (5) can be stated in terms of the other response spectrum parameters, as follows:

$$\bar{\alpha} \leq \sum | \alpha_n V_n / \omega_n | \qquad (6)$$

$$\bar{\alpha} \leq \sum | \alpha_n A_n g / \omega_n^2 | \qquad (7)$$

These relationships, of course, are applicable only to an elastic system.

The summations in the three preceding equations give absolute upper bounds to the response quantities. It is shown in Ref. 7, for example, that the probable value of the response parameter, α_p, is equal approximately to the square root of the sum of the squares of the modal values, as indicated by Eq. (8).

$$\alpha_{prob.} \approx \sqrt{\sum (\alpha_n D_n)^2} \qquad (8)$$

Equations corresponding to (8) can be written involving V_n, or $A_n g$, of course.

Along a line where D_n is constant, Eq. (5) assumes a simpler form. Similarly along lines where V_n or A_n are constant, Eqs. (6) and (7) assume also simpler forms. For example, if $D_n = D =$ constant, one has the result

$$\bar{\alpha} \leq D \sum | \alpha_n | \qquad (9)$$

If $\bar{\alpha}$ is set equal to D', then one has the result

$$\frac{D'}{D} \leq \sum | \alpha_n | \qquad (10)$$

Similarly one can compute V' and A' as follows, for the cases where V_n or A_n is constant, respectively:

$$\frac{V'}{V} \leq \sum \left(\frac{\omega_1}{\omega_n}\right) \left| \alpha_n \right| = \sum \left(\frac{f_1}{f_n}\right) \left| \alpha_n \right| \qquad (11)$$

$$\frac{A'}{A} \leq \sum \left(\frac{\omega_1^2}{\omega_n^2}\right) \left| \alpha_n \right| = \sum \left(\frac{f_1}{f_n}\right)^2 \left| \alpha_n \right| \qquad (12)$$

Equations (10), (11), and (12) give a procedure by which the multi-degree-of-freedom spectrum can be plotted from the single-degree spectrum values. A different spectrum will be drawn for each response quantity desired of the multi-degree-of-freedom system.

A number of comparisons have been made for the exact responses of multi-degree-of-freedom systems for various ground motions, compared with the computations of upper bounds from equations such as (5), (6), and (7), or of probable values from equations such as (8).

Such calculations are reported in Refs. 5, 6, and 8, for example. In Ref. 8 it is shown that for systems with a small number of degrees-of-freedom, say 4 or less, the true response for an earthquake motion is very nearly equal to but slightly less than the sum of the absolute values of the modal maxima. For a large number of degrees-of-freedom, say 12 or more, the true response is very nearly equal to the square root of the sum of the squares of the modal maxima. For systems with an intermediate number of degrees-of-freedom, the true response is generally about midway between these two values. In Figs. 15 and 16 there are shown calculations of response for two different 5 degree-of-freedom systems; in Fig. 15 the system corresponds to uniform masses and springs, and in Fig. 16, the system has varying springs but uniform masses, in both cases subjected to a particular ground shock, as recorded in event Aardvark, a nuclear detonation. The responses are given for the relative story deflections in each of the five stories of the 5 degree-of-freedom shear beam. Both the sum of the absolute values of the modal maxima and the square root of the sum of the squares are plotted in the figure, which compares the true spring distortion (computed for the actual ground motion input by a step by step integration procedure) with the approximate spring distortion computed from the true response spectrum. A value of s_a = 1.00s would correspond to the approximate value equal to the exact value. It is noted that in every case the sum of the absolute values of the modal maxima lies above this value, as it should, and the square root of the sum of the squares value lies below, although there is no reason why in some cases some of these values should not lie above the line s_a = 1.00s. The general rule described previously would indicate, for a 5 degree-of-freedom system, one should use the average of the two values so computed. It appears that this is borne out by the results plotted in Figs. 15 and 16 also. However, errors less than about 40 per cent in general, and in most cases not much greater than about 20 per cent would be involved if one were to use either of the two approximations for this particular case.

Similar calculations for other input motions and somewhat different structures lead to the same general conclusions, which appear to be valid except where the spacing of the frequencies is such that several modes having nearly the same frequency are involved.

It appears, therefore, that we can use the results of modal calculations to infer the responses for actual earthquakes, by use of the response spectrum. This may introduce errors in some cases, but the errors are relatively small. One must keep in mind the fact that the average of the responses for a large number of earthquakes would be more nearly consistent with the results arising from the use of a smooth spectrum value and would be more nearly representative of probable values, than the values for a particular single earthquake.

Calculations have been made for a number of cases of inelastic multi-degree-of-freedom systems to give their responses to particular earthquakes. Such calculations are reported in Ref. 5 (for two-degree-of-freedom systems), Refs. 9, 10 and 11 (for a five-degree system). Although generalizations cannot be readily made, because of the sparseness of the data, it does appear that the generalizations made herein, and outlined in Fig. 12, are applicable as bounds for a multi-degree-of-freedom system, in general, provided that the response spectra for the multi-degree-of-freedom system is determined as indicated in Fig. 14.

RESULTS OF ELASTIC ANALYSES FOR TALL BUILDINGS

Shears and overturning moments in a number of tall buildings have been computed and are reported in various references. A particularly interesting comparison is shown in Ref. 12. In using the data from Ref. 12, however, one should keep in mind the fact that the response spectrum used was one which corresponded to a maximum velocity response of about 1.2 in./sec. or roughly about one-sixteenth the El Centro response spectrum. The calculations when so interpreted indicate that the Uniform Building Code values for base shear are from 1/2 to 1/6 those computed by the more exact analysis, and that the shears at higher elevations of the building generally increase relative to those given by the Uniform Building Code. Overturning moments are also somewhat higher than those given by the Code, but the upper stories generally have a more conservative moment relative to the base value.

A new series of calculations is reported herein. These calculations were made by the writer's colleague, Dr. S.J. Fenves, using the high-speed digital computer in the Department of Computer Science at the University of Illinois. Calculations were made for a series of buildings having 40, 30, 20 and 10 stories, with parameters chosen so that the fundamental periods of vibration for all of these were either 3 secs. or 1 sec. The response spectrum used was one having a displacement bound of 10 in., a velocity bound of 20 in./sec., and an acceleration bound of 0.667g. This is very nearly the response spectrum for the El Centro earthquake for elastic conditions. Two types of buildings were considered, namely, a flexural building, corresponding to a shear wall structure of uniform properties over the height, and a "shear beam" building, corresponding to a frame structure. Deflections of the flexural building are shown in Fig. 17. Values are given for the square root of the sum of the squares, designated "RMS" on the figure and for the sum of the absolute values of the modal maxima, designated by "MAX" on the figures.

The maximum story shears are shown in Fig. 18. The Uniform Building Code values, for a coefficient K=1, but multiplied by factor of 1.5 to account for the difference between working stress and yield point, are shown on the figure for comparison. It is seen that the base shear computed for these buildings is about 3 to 3.5 times the Uniform Building Code value. Near the top of the building, the values are from 5 to 6.5 times the Uniform Building Code values. These comparisons are for the "RMS" values. The "MAX" values are considerably higher, but are not considered reasonable to use for the multi-story building. This is an indication that for the building described, and for the El Centro Earthquake, a ductility factor of the order of 3 is required at the base, and about 5 near the top of the building, in order that the Uniform Building Code lead to a design which is not inadequate. Unless these ductility factors are provided for by the details of construction and inspection, an earthquake of intensity equal to the El Centro Earthquake would produce serious consequences in the building considered.

A similar comparison is made for overturning moments. Here the comparisons are made against the cantilever moments rather than the design overturning moments by the Uniform Building Code. Near the base of the building, the "RMS" values are 1.6 to 1.8 times the Uniform Building Code values for overturning moments. These are about one-half the values, relatively, of the base shears and correspond to a reduction factor for overturning moment of the order of about 50 per cent of the cantilever moment values corresponding to the "RMS" base shears. For comparison, the shear beam building for the same heights and periods leads to results that are given in Figs. 20, 21, and 22. The deflections shown in Fig. 20 are slightly less than those shown in Fig. 17, but the shape of the deflection curve is considerably different. The maximum story shears, as shown in Fig. 21, indicate a ductility factor requirement of 2 or slightly less at the base, and about 2.5 to 4 near the top of the building. Somewhat lower ductility factors are required, therefore, for the frame or shear beam building, for the same conditions, relative to the flexural or shear wall building. The overturning moment values are 1.7 to 1.8 times the Uniform Building Code cantilever moments, which corresponds to a reduction factor of the order of about 0.85 to 0.9, instead of 0.5. In other words, the shear beam building or frame, has a much greater overturning moment, relatively, than the flexural or shear wall building.

A further comparison still is given for the shear beam building for the shear beam building for a period of 1.0 sec., for the same number of stories. This is, of course, an unreasonable type of structure even to consider for 40 stories, but not for 10 stories. However, it is of intrest to note that the ductility factors required for shear for these buildings range from about 3.7 to 4 at the base, and are about the same at the top, although for the 10 story building they do go up to slightly larger values. The reduction factor for overturning moment is about 0.95 to 1.0, however, or in other words the overturning moment is nearly equal to the cantilever moment.

Based on these results and other analytical studies, it is concluded that a reasonably conservative design basis for a building involves a response spectrum approach, but with the use of a reduced ground motion, corresponding to a selected value of ductility factor which can be mobilized by the method of construction chosen. The method of selecting the response spectrum to use in such an analysis is indicated in Fig. 26. The trapezoidal set of lines designated by the legend "ground motion" corresponds to the maximum values of ground displacement, velocity, and acceleration. The elastic spectrum, designated by the symbol $\mu = 1$, for displacement and acceleration, D and A, represents slightly amplified values, corresponding to an elastic response spectrum for the ground motion considered. The curve marked D for $\mu = 5$, is the displacement spectrum for a ductility factor of 5, and the curve marked A for $\mu = 5$ is the acceleration or force spectrum for the same conditions. These are drawn so as to conserve displacement on the left-hand side, force on the right-hand side, and energy in the central part, An elastic analysis made for the reduced acceleration spectrum would, therefore, correspond to the ductility values derived for the conditions described.

The relations between the various bounding lines in Fig. 26, for an elasto-plastic resistance function, are relatively simple to compute. A summary of the values so computed is given in Table I. The table lists, for each of the quantities that can be conserved, namely, displacement, energy or velocity, and force or acceleration, the ratio between the elasto- plastic and the purely elastic response values for total displacement or for acceleration, as a function of the ductility factor μ.

For example, when $\mu = 5$, the tabulated values indicated are as follows: along a constant displacement line, the displacement is the same, and the acceleration is one-fifth as much for the elasto-plastic spectrum as for the elastic spectrum. Along a constant velocity line, the displacement is five-thirds as great, and the acceleration one-third as great, for the elasto-plastic spectrum compared with the elastic spectrum. Finally, along a line of constant acceleration, the displacement is five times as great and the acceleration value is the same as the value for elastic response.

DESIGN OF COMPOSITE 41-STORY BUILDING

An example of the use of these concepts in the design of an actual building is illustrated next. The particular building chosen was designed originally as a shear wall structure. However, in reviewing the design, I felt that it was desirable to add more flexibility in the lower stories, and recommended that these be made of steel frame construction so as to permit greater flexibility and energy absorbing capacity there. The lower six stories were, therefore, made of steel frame construction and the upper part was to be designed to be either a concrete shear wall or a braced steel frame. The deflection for the composite building, for a Zone 2 earthquake response spectrum, is shown in Fig. 27. This is on the upper bound of Zone 2, and is about three-quarters of the El Centro earthquake response spectrum. The building has a period of about 2.3 secs.

The maximum story shears are shown in Fig. 28. For the response spectrum selected, the ductility factor required is about 2 or slightly less. Indicative of this is the comparison shown in Fig. 29, where the spectrum is one which corresponds to a ductility factor of about 2 relative to that used in Fig. 28. Here, the design values are slightly less than those corresponding to the Uniform Building Code, when the latter is increased by the ratio of yield point to working stress values. It is concluded that this building will be adequate for a ductility factor of 2, for the exposure considered, namely, about three-quarters of El Centro, and will have adequate resistance to cope with a stronger earthquake without danger of collapse. The purpose of selecting a relatively conservative ductility factor of 2 in this case was to avoid expensive repairs for the expected earthquake hazard.

The overturning moments for the same building are shown in Fig. 30. It is noted that the overturning moment is considerably higher than that given by the Uniform Building Code, and the design was made so as to be consistent with the computed values rather than with the Code values. The overturning moment corresponding to the reduced spectrum is shown in Fig. 31. It appears that the reduction in cantilever moment corresponding to the Uniform Building Code value for this building is almost twice as great as from the analysis.

SPECIAL CONSIDERATIONS

In regions where unusual types of ground motions can be expected because of oscillations of the soil over deeply buried rock, modifications to response spectrum must be considered. This is particularly essential in places like Mexico City where amplification of ground motions in the range of periods from 2 to 2.5 secs. occurs because of the natural frequency of the bowl of soft soil on which most of Mexico City is founded. An example of a building designed for the special conditions in Mexico City is given in Ref. 13. This building, the Latino Americana Tower, was designed for a base shear of the order of 500 metric tons, corresponding to an earthquake of Modified Mercalli Intensity VIII, but taking into account the amplification of motions corresponding to the natural period of vibration of the soil on which the building rests. During the construction the building was modified to provide for a heavy television tower at the top. Shortly after the construction was completed, a major earthquake occurred, corresponding to the intensity for which the building was designed. Recording instruments had been installed in the building, on which records were obtained during the earthquake. The values recorded were almost precisely those which had been considered in the design as being consistent with the probable values corresponding to Eq. (8) herein.

In regions where substantial vertical earthquake accelerations occur, or in some cases for unusual types of construction which may have a different resistance to motion in one direction than another, a peculiar phenomenon similar to "pumping" may be encountered. This is best illustrated by an analysis described in Ref. 14. Consider, for example, a mass sliding under a constant force, having a frictional resistance against the sliding surface. The friction coefficient may be characterized by the coefficient N. If one considers the effect of the gravity action on the sliding mass, the force required to produce downhill sliding is less than that required to produce uphill sliding. Two extreme cases of rigid-plastic resistance, corresponding to the condition shown in Fig. 32, are considered in the calculations described in the following. These involve a symmetrical resistance corresponding to a mass sliding horizontally, with the same resistance in either direction, and an unsymmetrical resistance, corresponding to a frictional resistance against sliding in one direction and an infinite resistance against sliding in the other direction.

Calculations were made for these two conditions for the four earthquakes described in Table II. For these four earthquakes, the values of acceleration and time scale were modified so as to give a maximum acceleration for all of the earthquakes of 0.5g, and a maximum ground velocity of 30 in./sec. The maximum displacements for the four earthquakes range from 20.5 to 51.2 in. The purpose of normalizing the earthquakes was to obtain a more consistent set of data for strong earthquake conditions.

The case of symmetrical resistances is shown in Fig. 33, where the maximum displacement of the sliding mass relative to its support is plotted against the ratio of the resistance coefficient to the maximum acceleration of 0.5g. The envelope of the plotted points corresponds to a condition in which the maximum energy of the mass, corresponding to the quantity $1/2\ MV^2$, is absorbed by the resistance multiplied by the distance over which sliding occurs. A slight correction to this energy, to account for the fact sliding does not occur unless the acceleration exceeds the resistance, is indicated by the lower curve. In no case does the maximum displacement exceed the maximum ground displacement for the condition of symmetrical resistance.

Figure 34 summarizes the displacement for unsymmetrical resistance. Here the displacements are almost 6 times as great as those in Fig. 33, for the maximum condition. In other words, the earthquake considered corresponds to a condition of something like 6 pulses for unsymmetrical resistance contrasted with only 1 effective pulse for symmetrical resistance contrasted with only 1 effective pulse for symmetrical resistance. The factor 6 is no doubt dependent upon the duration of the earthquake motions. It is probably proportional to the square root of the total duration. This factor is consistent with a duration of about 30 to 40 secs., corresponding to the duration of the earthquakes for which the calculations were made.

Similar calculations were made with elasto-plastic resistances having different yield points in the two directions. As soon as the difference in yield point was more than just nominal in value, the "pumping" appeared to be almost as great as for the rigid plastic resistance reported in Figs. 33 and 34. Hence, it is concluded that substantial increases in displacement can occur under conditions where yielding occurs for vertical motions, or where the conditions of yielding or of coupling with adjacent structures introduces a substantially greater resistance to deflection in one direction than in another. Care must be taken to avoid such conditions or to provide adequately for them.

In buildings which have a combination of a frame and a shear wall to resist earthquakes, consideration must be given to the interaction of the different types of construction, their different ductilities and flexibilities. The difference in pattern of displacement of a frame and a shear wall is shown in Fig. 35. When a building contains these two elements, the partition of shear between them must be such as to produce equal deflections of the two elements. Because of the difference in the shape of curves, it appears that the shear wall will take more than the total shear near the base, but will be restrained relatively in the opposite direction by the frame in the upper part of the structure. Provision for the inter-reaction between the two elements must be made if the structure is to behave properly. Moreover, consideration of the change in configuration and energy absorbing capacity, if the shear wall fails during the course of the deflection of the structure, must be taken into account in assessing the over-all behavior of the composite structure.

CONCLUDING REMARKS

For earthquakes of the intensities experienced in regions subjected to strong earthquakes, energy absorbing capacity and ductility are essential to permit deformations to occur beyond the range of linear behavior or the range of ordinary working stresses. It is possible to design modern tall buildings to resist earthquakes with an adequate margin of safety. The margin that can be achieved is a function of the price one is willing to pay as a sort of insurance against normally expected earthquake intensities, and the degree of damage one is willing to permit in an extraordinarily severe earthquake.

The general philosophy is proposed that a building should suffer little if any damage in the intensity of earthquake that might normally be expected several times during its life, in order to avoid expensive repairs. However, the building should have an adequate reserve capacity against collapse should an extreme earthquake occur at any time.

REFERENCES

1. J. A. Blume, N. M. Newmark, and L. H. Corning, "Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motions," Portland Cement Association, Chicago, Illinois, 1961.

2. N. M. Newmark, and A. S. Veletsos, "Design Procedures for Shock Isolation Systems of Underground Protective Structures, Vol. III, Response Spectra of Single-Degree-of-Freedom Elastic and Inelastic Systems," Report for Air Force Weapons Laboratory, by Newmark, Hansen and Associates under subcontract to MRD Division, General American Transportation Corporation, RTD TDR 63-3096, Vol. III.

3. A. S. Veletsos, N. M. Newmark, and C. V. Chelapati, "Deformation Spectra for Elastic and Elasto-Plastic Systems Subjected to Ground Shock and Earthquake Motions," Proceedings Third World Conference on Earthquake Engineering, New Zealand, 1965.

4. A. S. Veletsos and N. M. Newmark, "Effect of Inelastic Behavior on the Response of Simple Systems to Earthquake Motions," Proceedings Second World Conference on Earthquake Engineering, Tokyo, 1960, Vol. II, p. 895-912.

5. N. M. Newmark, W. H. Walker, A. S. Veletsos, and R. J. Mosborg, "Design Procedures for Shock Isolation Systems of Underground Protective Structures, Vol. IV, Response Spectra of Two-Degree-of-Freedom Elastic and Inelastic Systems," Report for Air Force Weapons Laboratory, by Newmark, Hansen and Associates, under subcontract to MRD Division, General American Transportation Corporation, RTD TDR 63-3096, Vol. IV.

6. N. M. Newmark, W. H. Walker, A. S. Veletsos, and R. J. Mosborg, "Design Procedures for Shock Isolation Systems of Underground Protective Structures, Vol. V, Response Spectra of Multi-Degree-of-Freedom Elastic Systems," Report for Air Force Weapons Laboratory by Newmark, Hansen and Associates under subcontract to MRD Division, General American Transportation Corporation, RTD TDR 63-3096, Vol. V.

7. L. E. Goodman, E. Rosenblueth, and N.M. Newmark, "Aseismic Design of Firmly Founded Elastic Structures," Transactions ASCE, Vol. 120, 1955, p. 782-802.

8. R. L. Jennings and N. M. Newmark, "Elastic Response of Multi-Story Shear-Beam-Type Structures Subjected to Strong Ground Motion," Proceedings Second World Conference on Earthquake Engineering, Tokyo, 1960, Vol. II, p. 699-718.

9. J. Penzien, "Elasto-Plastic Response of Idealized Multi-Story Structures Subjected to a Strong Motion Earthquake, "Proceedings Second

World Conference on Earthquake Engineering, Tokyo, 1960, Vol. II, p. 739-760.

10. R. W. Clough, K.L. Benuska, and E. L. Wilson, "Inelastic Earthquake Response of Tall Buildings," Proceedings Third World Conference on Earthquake Engineering, New Zealand, 1965.

11. J. Heer, "Response of Inelastic Systems to Ground Shock," Ph.D. Dissertation, University of Illinois, 1965.

12. J. I. Bustamante, "Seismic Shears and Overturning Moments in Buildings," Proceedings Third World Conference on Earthquake Engineering, New Zealand, 1965.

13. L. Zeevaert and N. M. Newmark, "Aseismic Design of Latino Americana Tower in Mexico City," Proceedings World Conference on Earthquake Engineering, Berkeley, California, 1956, pp. 35-1 to 35-11.

14. N. M. Newmark, Fifth Rankine Lecture, "Effects of Earthquakes on Dams and Embankments," Geotechnique, Institution of Civil Engineers, London, 1965, p. 139-160.

TABLE I

RATIOS OF ELASTO-PLASTIC TO ELASTIC RESPONSE SPECTRUM VALUES IN VARIOUS RANGES

Quantity Conserved	Elasto-Plastic Relative to Elastic Response	
	Total Displacement	Acceleration
Displacement	1	$1/\mu$
Energy or Velocity	$\mu/\sqrt{2\mu - 1}$	$1/\sqrt{2\mu - 1}$
Force or Acceleration	μ	1

TABLE II

EARTHQUAKE CONSIDERED IN ANALYSIS

Earthquake	Maximum Ground Motions				Normalized*
	Acceleration g	Velocity in./sec.	Displacement in.	Duration sec.	Displacement in.
1. Ferndale 12/21/54, N45E	0.205	10.5	8.26	20	27.7
2. Eureka 12/21/54, S11W	0.178	12.5	10.0	26	51.2
3. Olympia 4/13/49	0.210	8.28	9.29	26	20.5
4. El Centro 5/18/40, N-S	0.32	13.7	8.28	30	25.5

* Normalized to give acceleration = 0.50g and velocity = 30 in./sec.

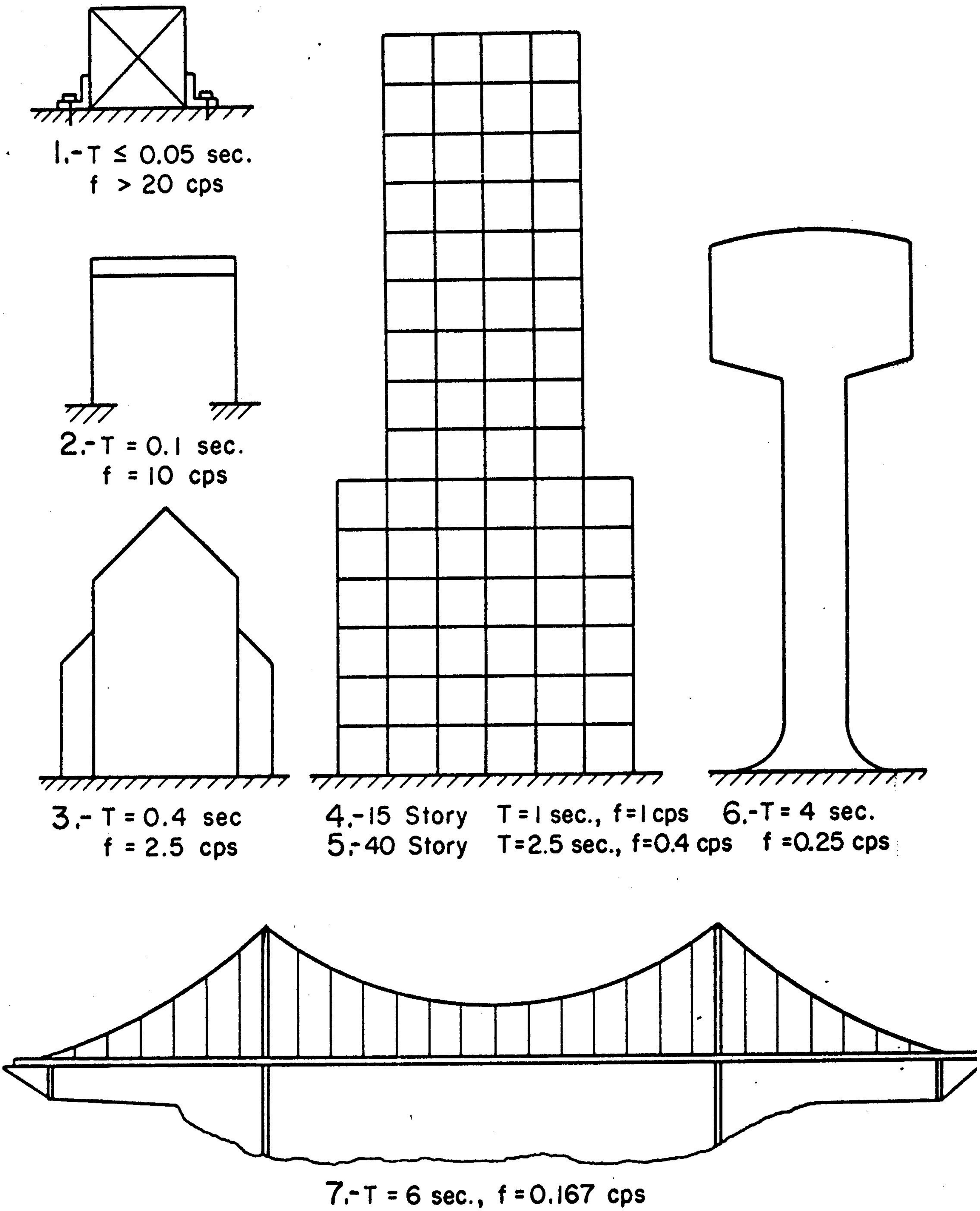


FIG. 1 STRUCTURES SUBJECTED TO EARTHQUAKE GROUND MOTIONS

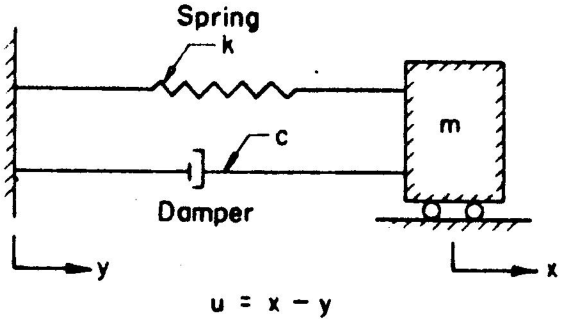


FIG. 2 SYSTEM CONSIDERED

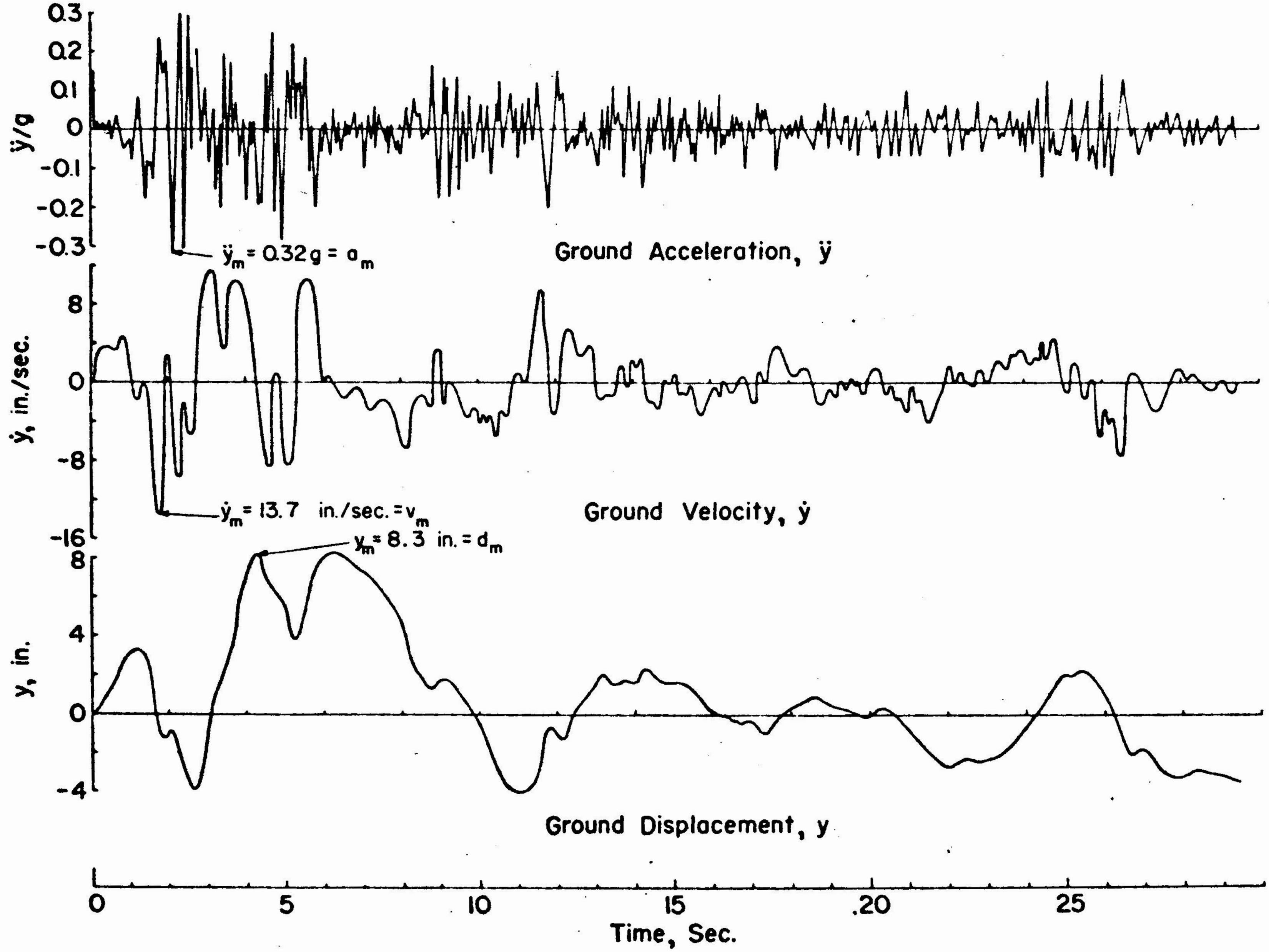


FIG. 3 EL CENTRO, CALIFORNIA EARTHQUAKE OF MAY 18, 1940, N-S COMPONENT

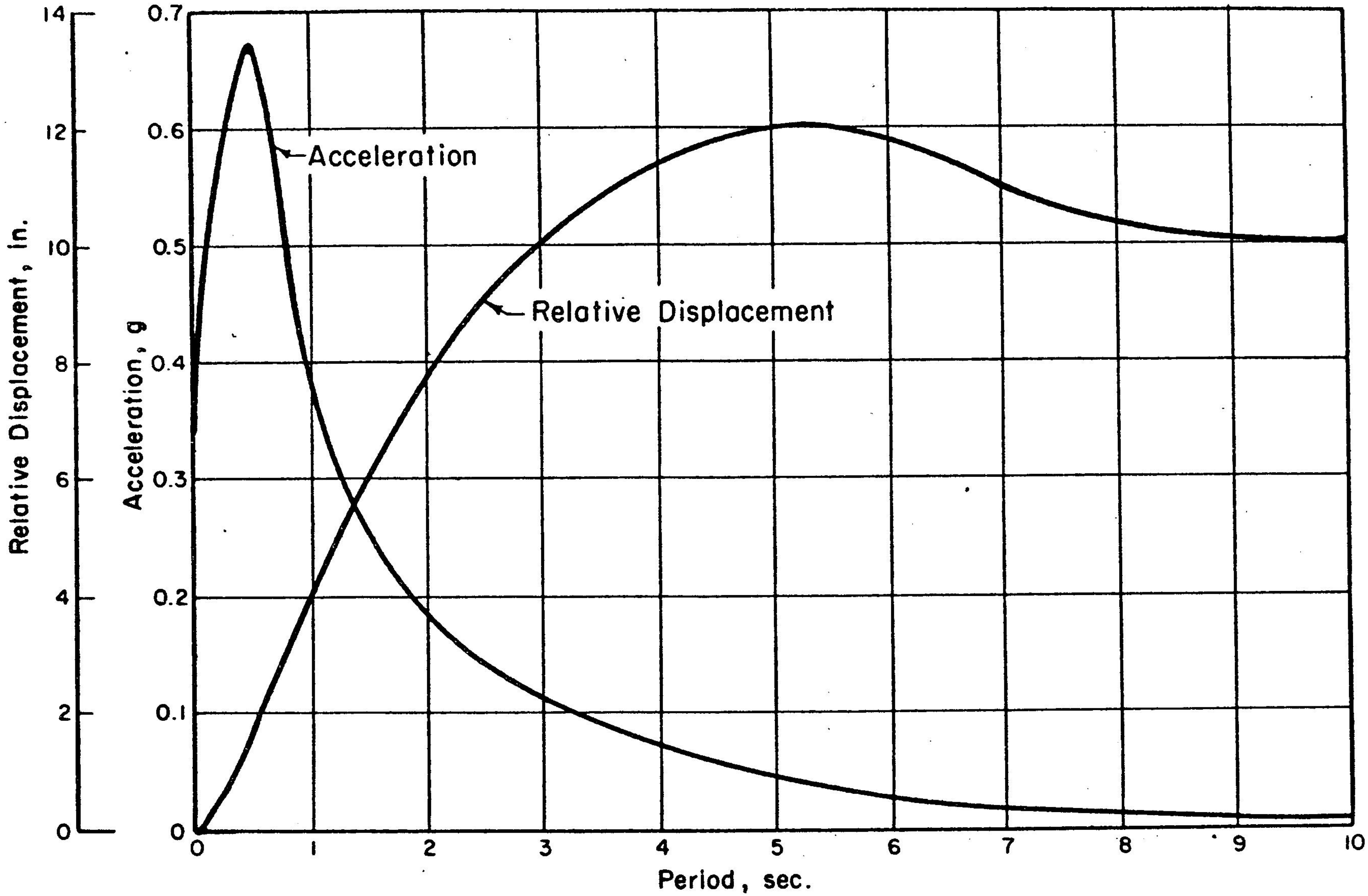


FIG. 4 ARITHMETIC PLOTS OF RESPONSE

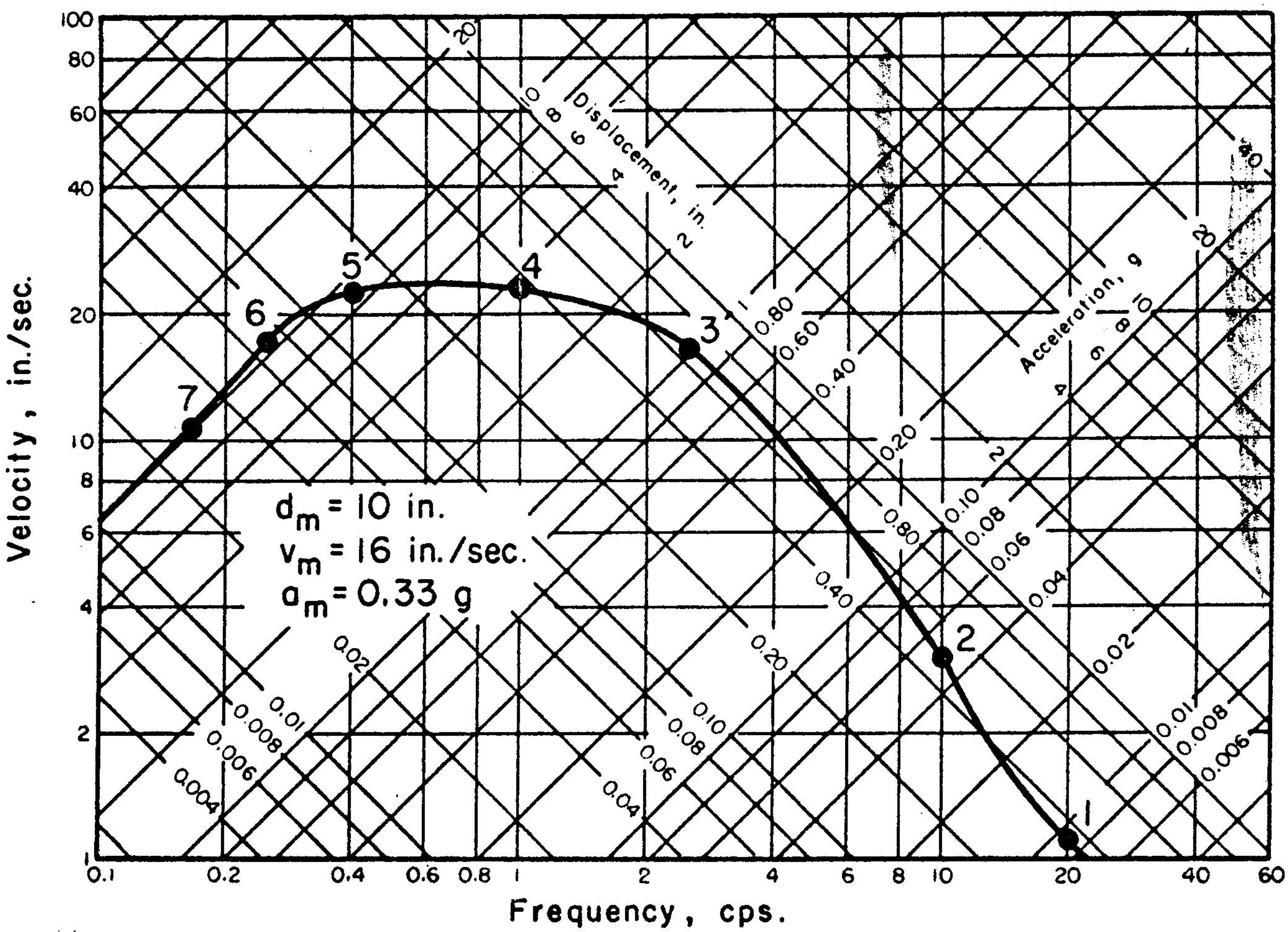


FIG. 5 RESPONSE SPECTRUM FOR TYPICAL EARTHQUAKE

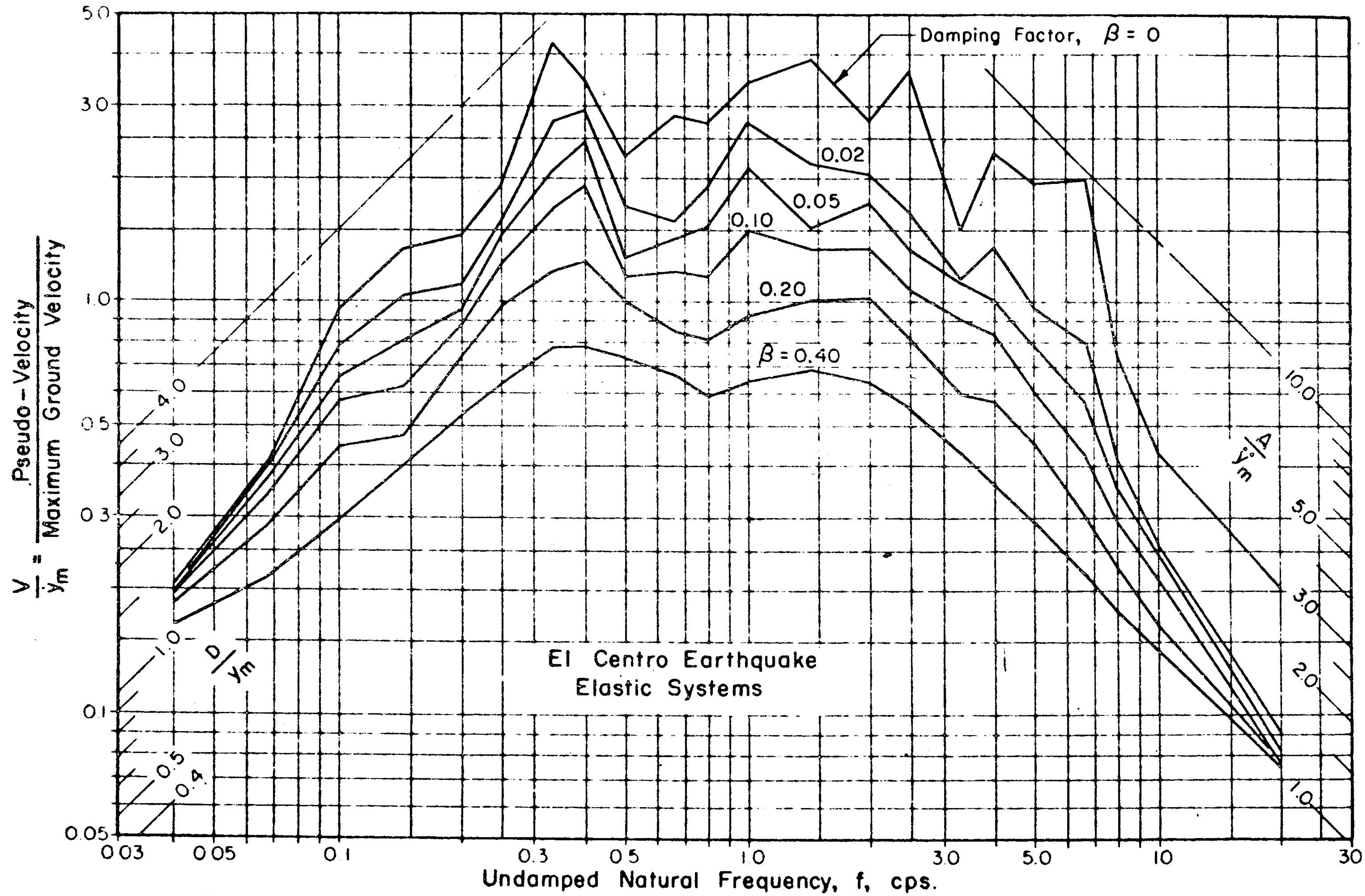


FIG. 6 DEFORMATION SPECTRA FOR ELASTIC SYSTEMS SUBJECTED TO THE EL CENTRO QUAKE

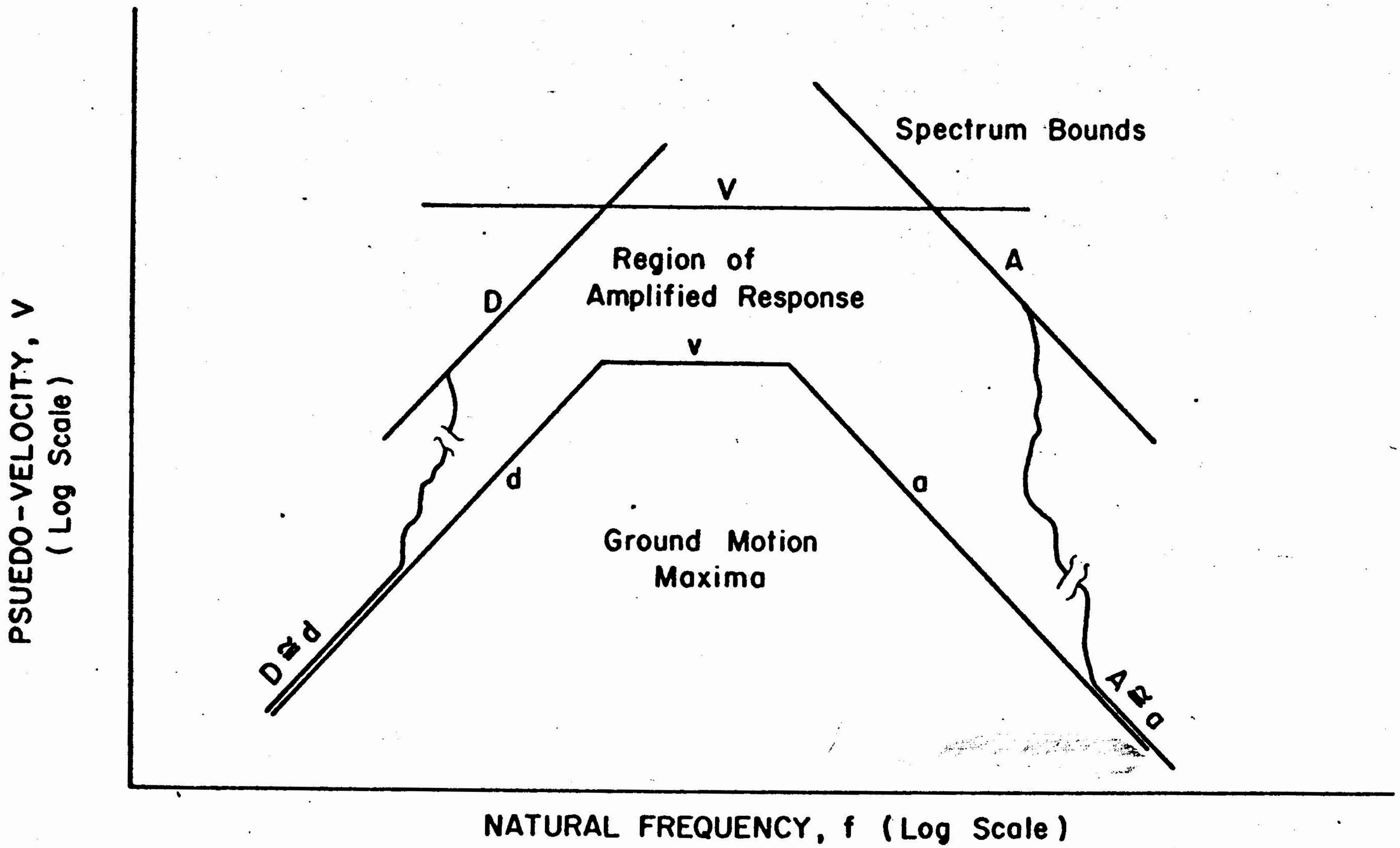


FIG. 7 TYPICAL TRIPARTITE LOGARITHMIC PLOT OF RESPONSE SPECTRUM BOUNDS COMPARED WITH MAXIMUM GROUND MOTIONS

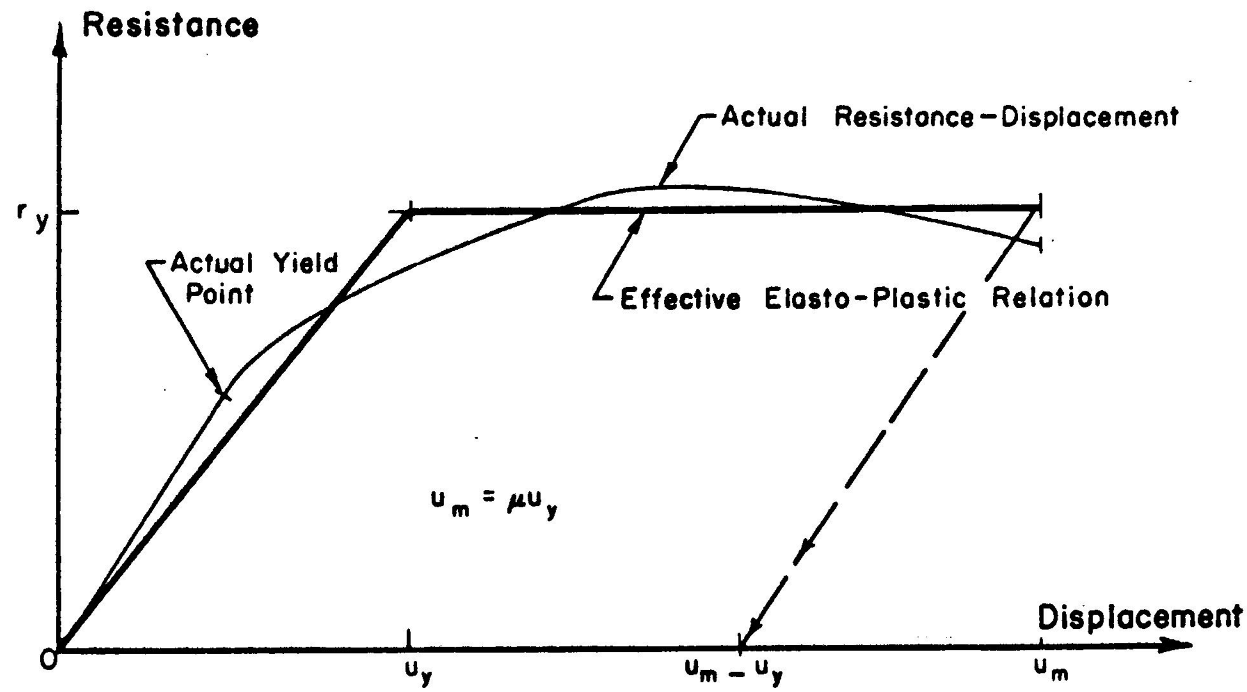


FIG. 8 RESISTANCE — DISPLACEMENT RELATIONSHIP

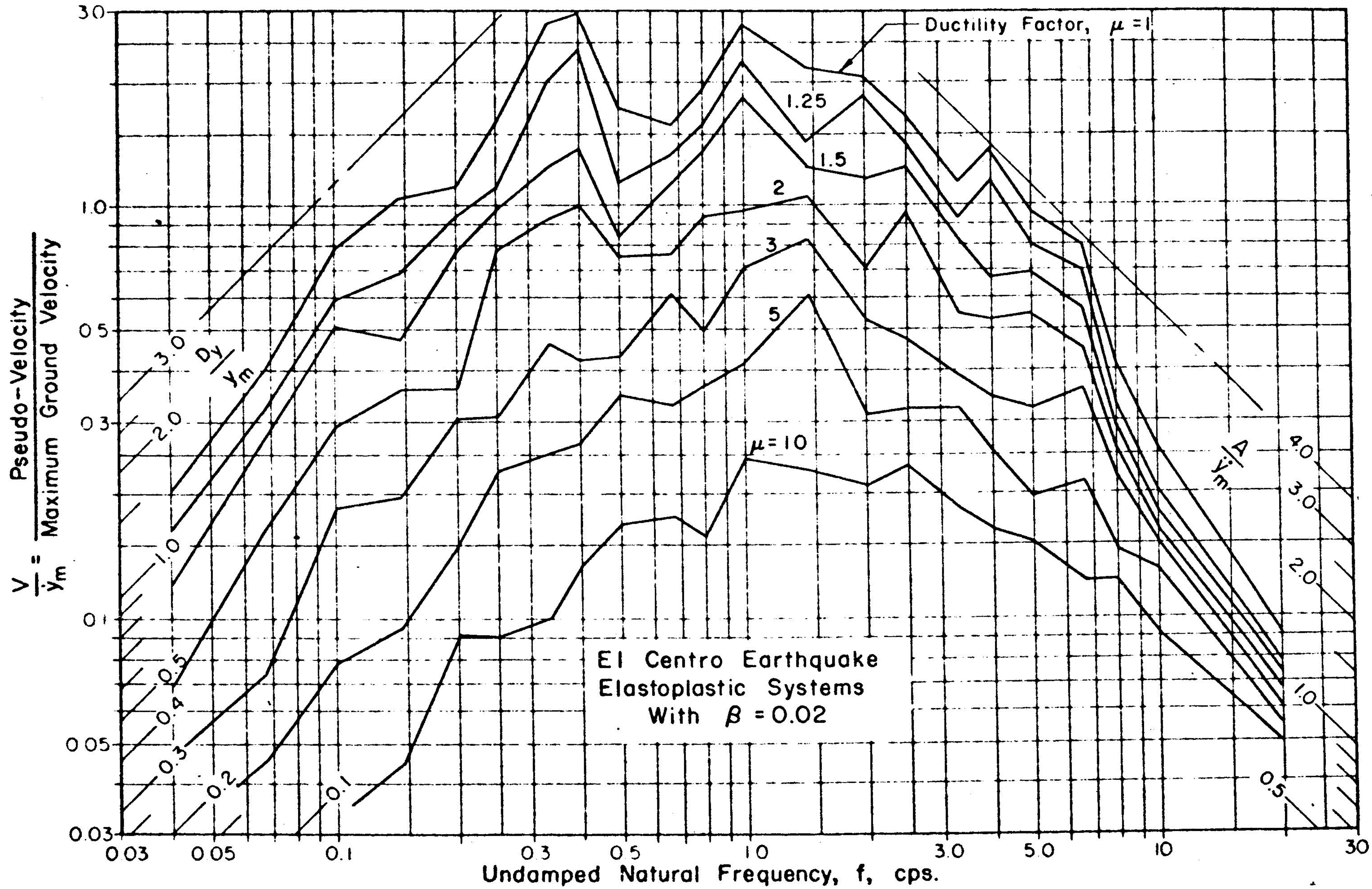


FIG. 9 DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS WITH TWO PERCENT CRITICAL DAMPING SUBJECTED TO THE EL CENTRO QUAKE

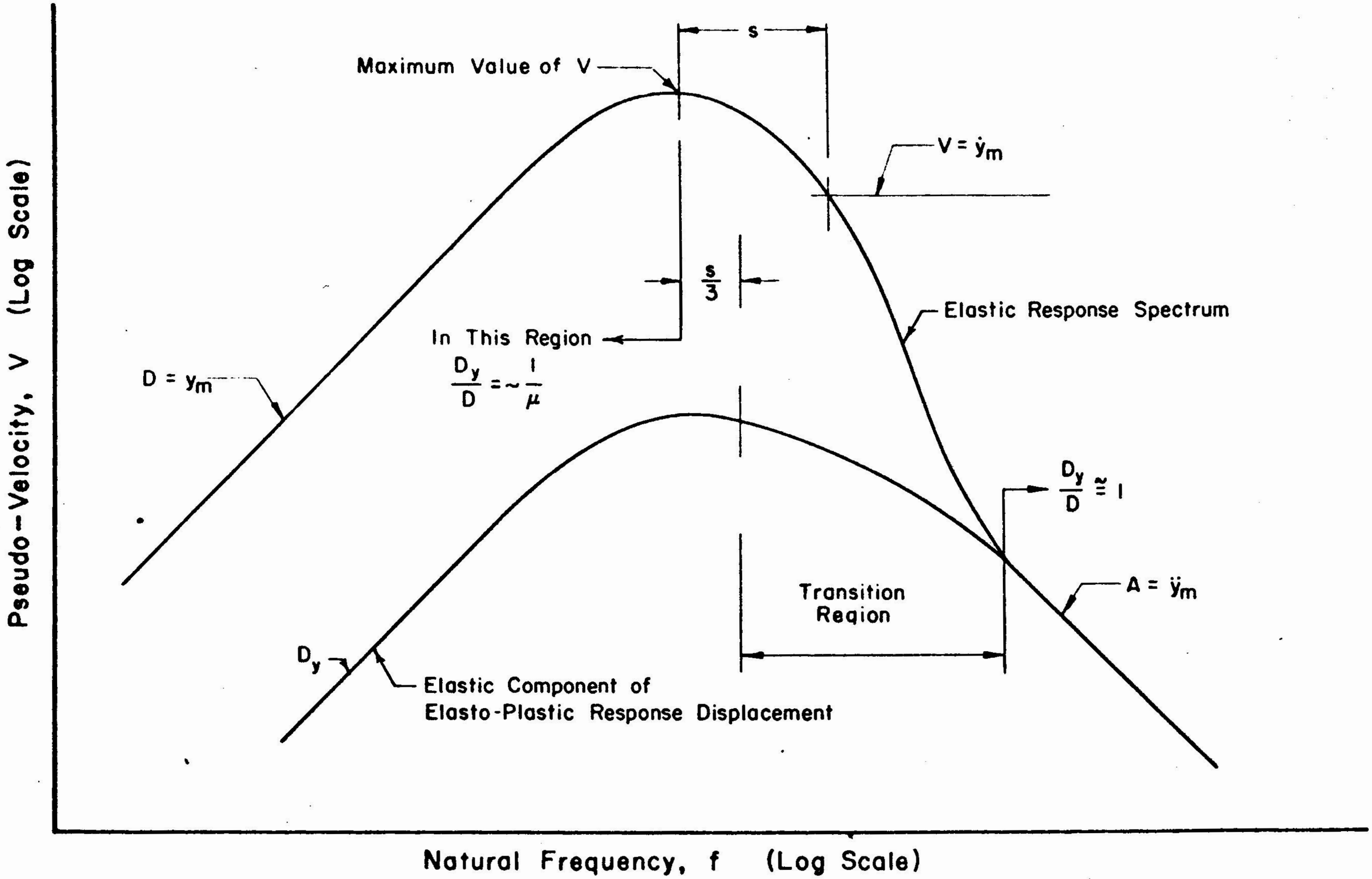


FIG. 10 APPROXIMATE DESIGN RULE FOR CONSTRUCTION OF DEFORMATION SPECTRA FOR ELASTOPLASTIC SYSTEMS

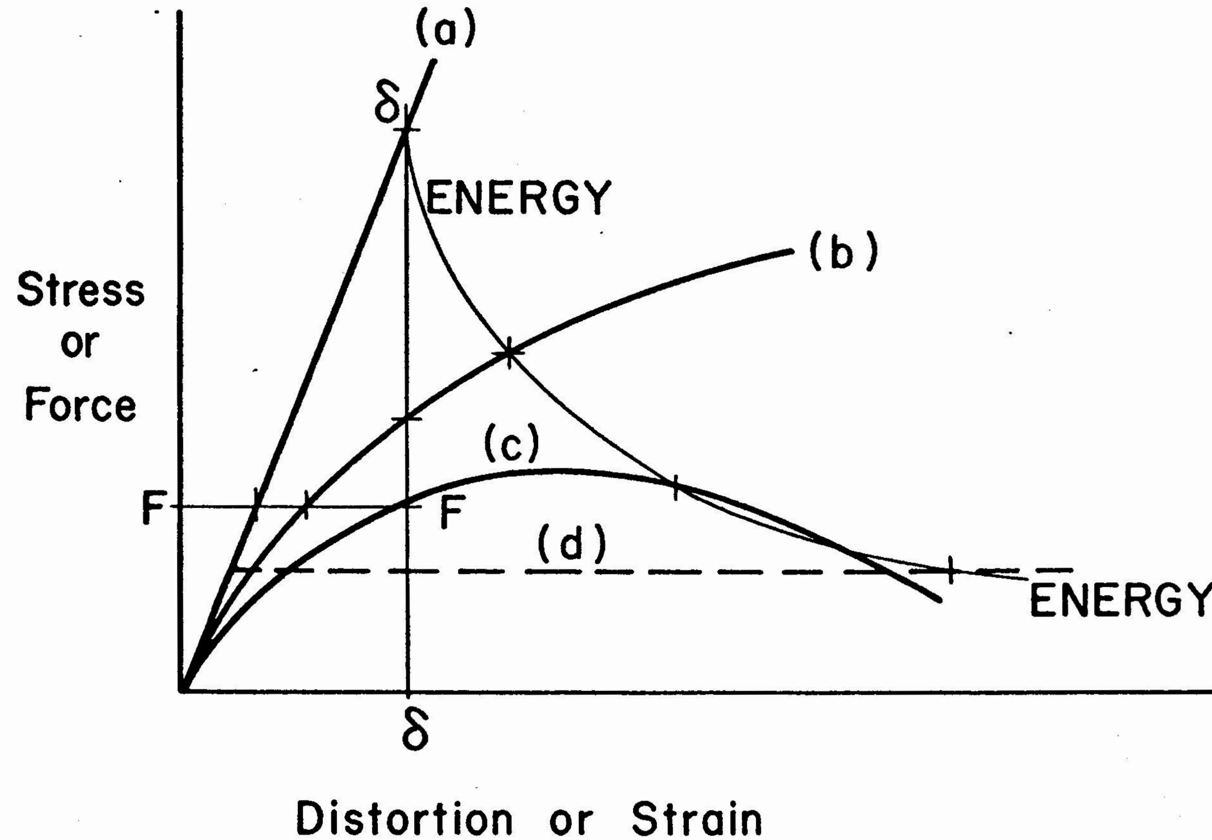


FIG. 11 COMPARISON OF STRAINS FOR EQUAL DISPLACEMENT, ENERGY, OR FORCE

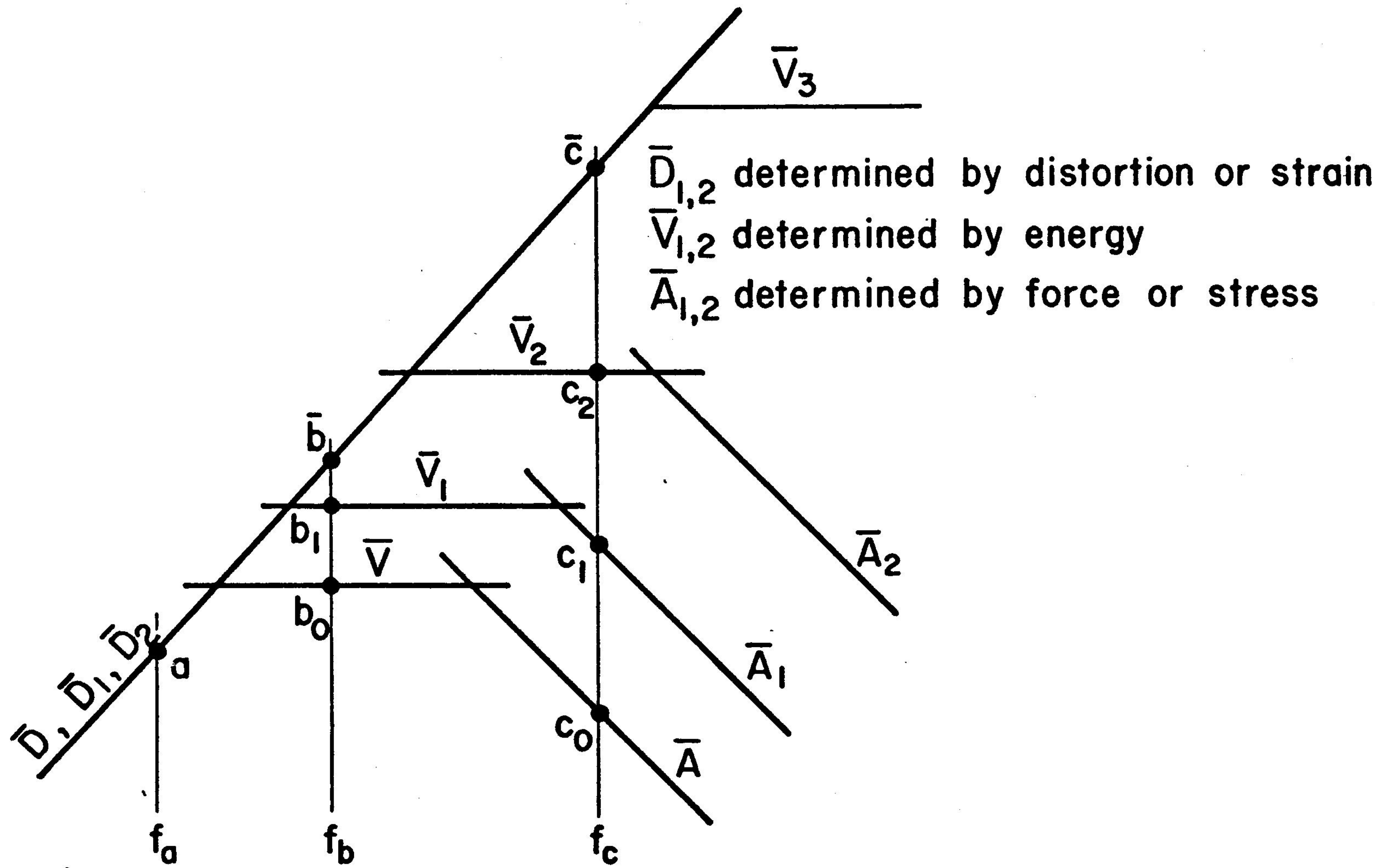
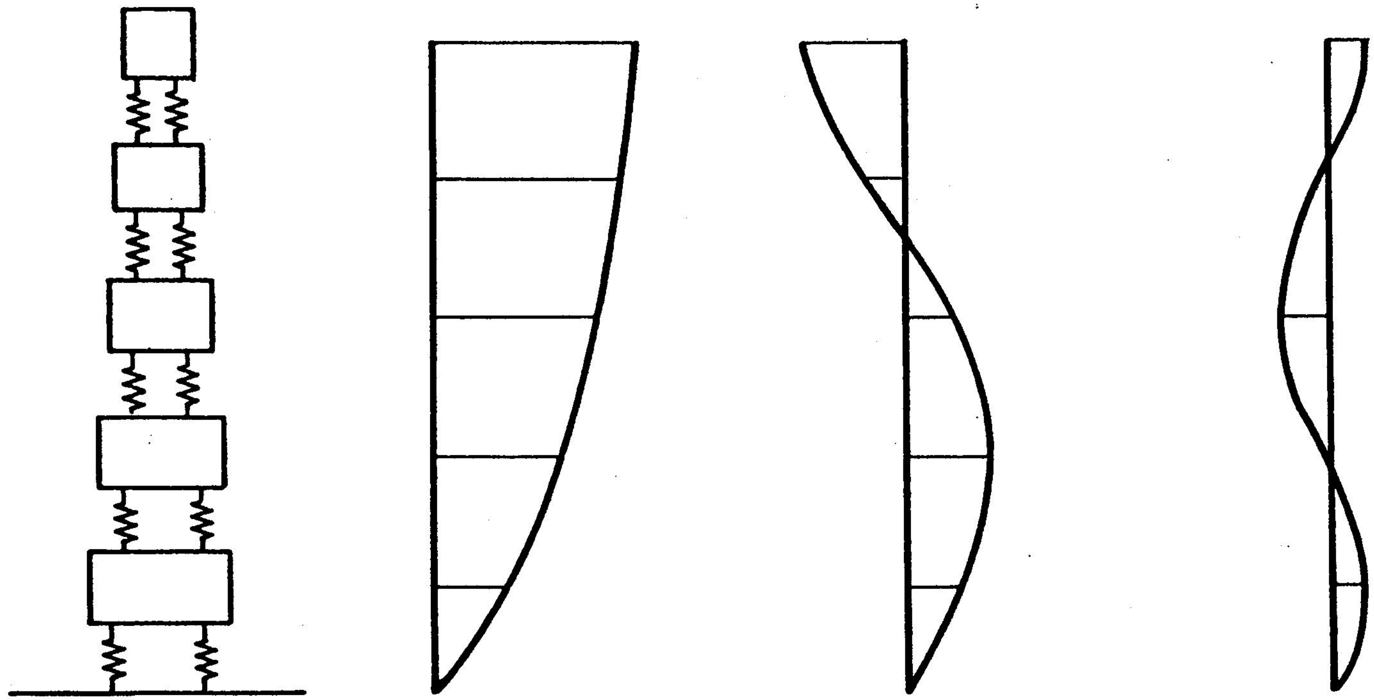


FIG. 12 RESPONSE SPECTRUM DISPLACEMENT LIMITS



(a)
Multi-Degree-of-Freedom System

(b)
Fundamental Mode

(c)
Second Mode

(d)
Third Mode

FIG. 13 MODES OF VIBRATION OF SHEAR BEAM

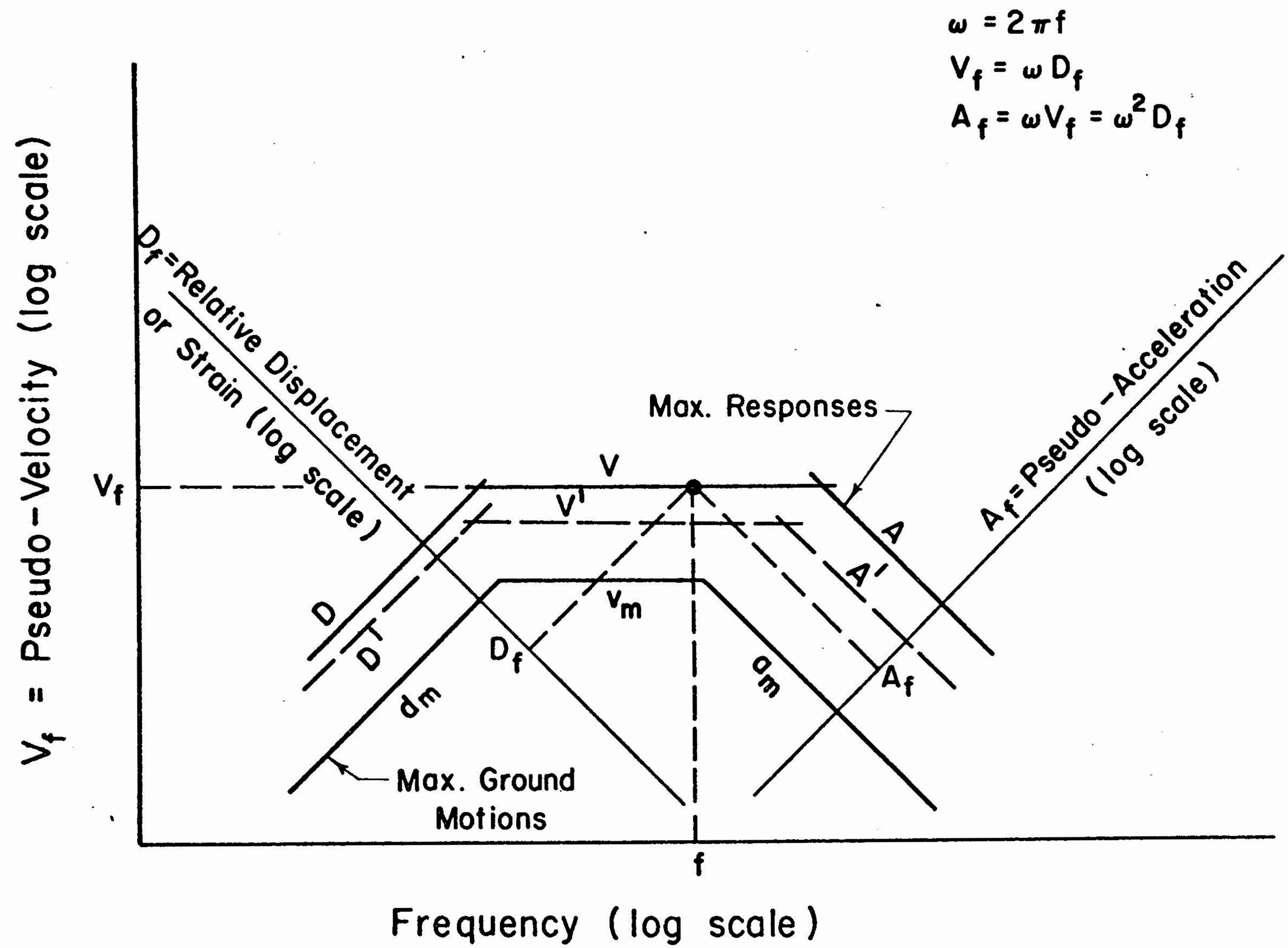


FIG. 14 TRIPARTITE LOGARITHMIC RESPONSE SPECTRUM PLOT FOR SINGLE AND MULTI-DEGREE OF FREEDOM SYSTEMS

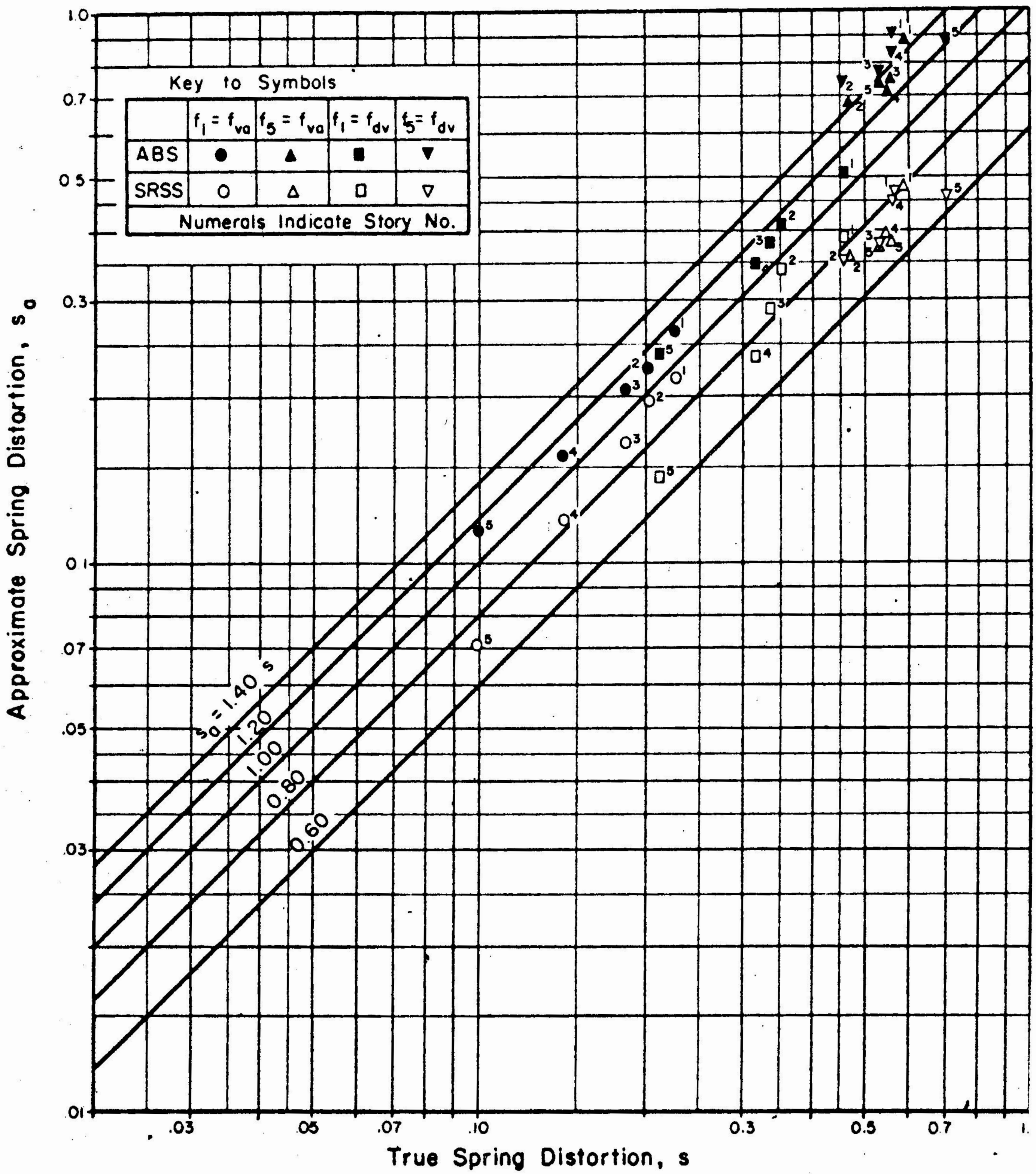


FIG. 15 COMPARISON OF MODAL AND EXACT SOLUTIONS (MODAL SOLUTIONS FOR TRUE SPECTRA) SYSTEM 5A; GROUND SHOCK FROM NUCLEAR DETONATION, EVENT AARDVARK

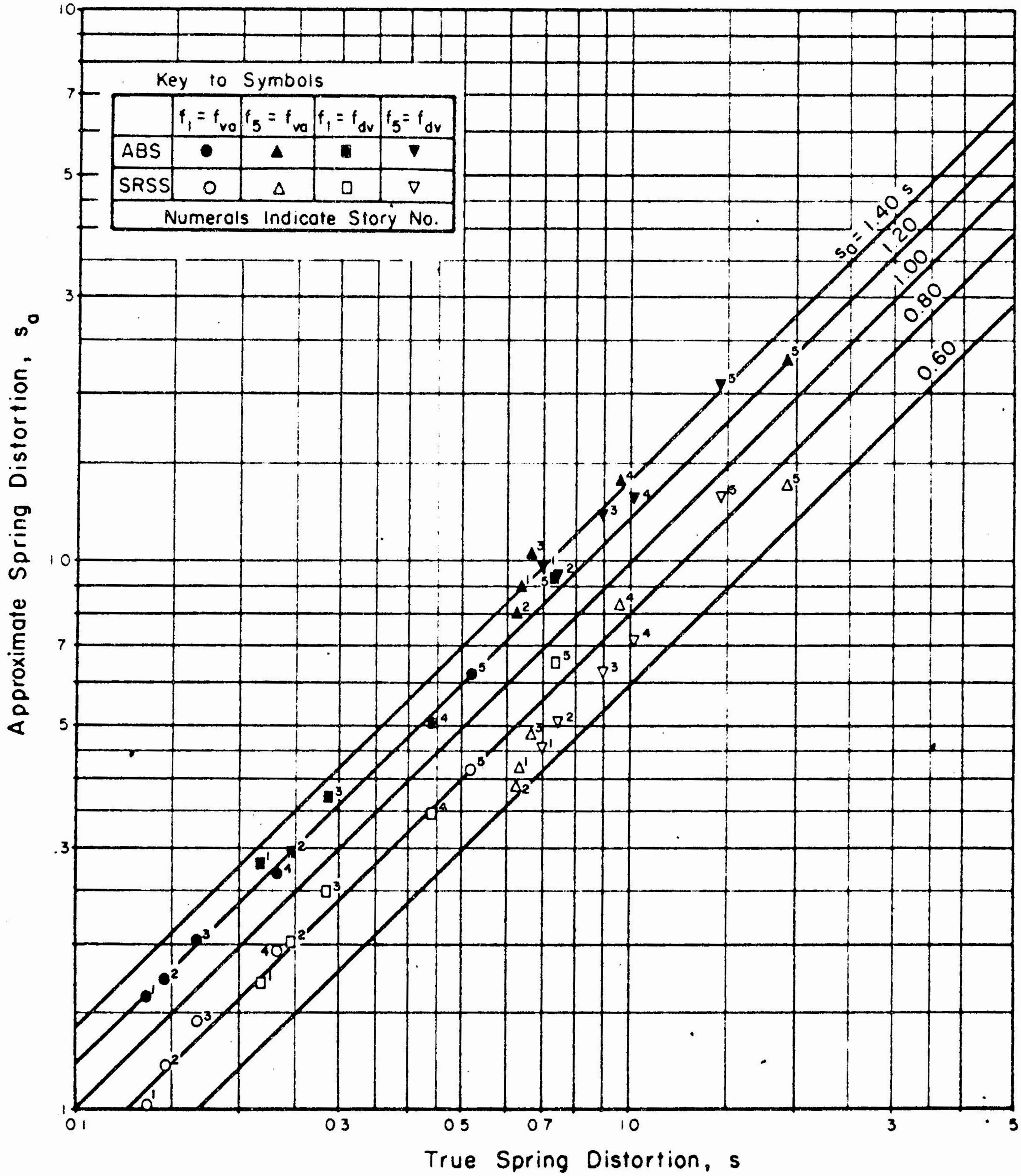


FIG. 16 COMPARISON OF MODAL AND EXACT SOLUTIONS (MODAL SOLUTIONS FOR TRUE SPECTRA) SYSTEM 5D; GROUND SHOCK FROM NUCLEAR DETONATION, EVENT AARDVARK

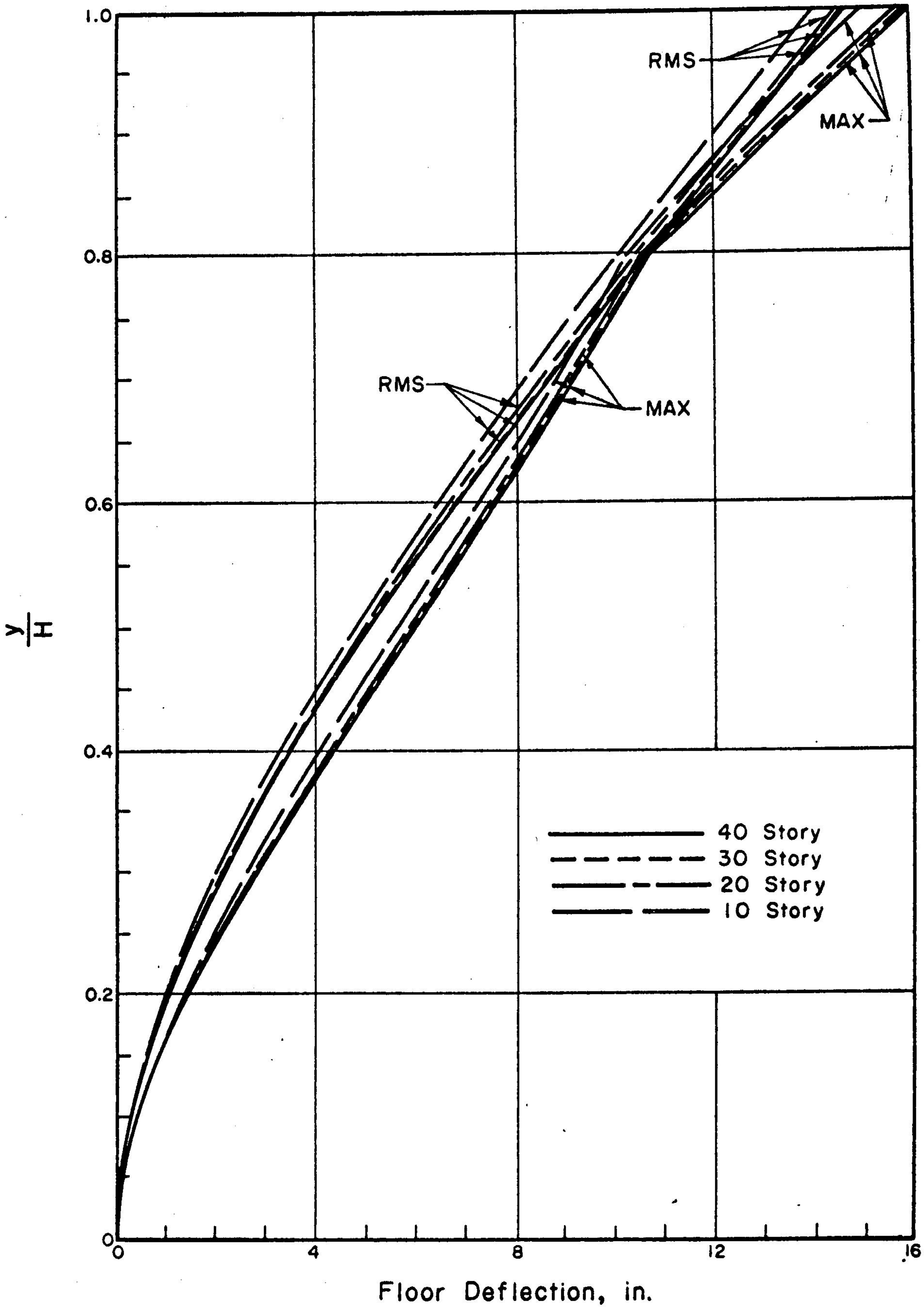


FIG. 17 MAXIMUM FLOOR DEFLECTIONS — 40-, 30-, 20-, AND 10-STORY FLEXURAL BUILDINGS, T_0 = 3 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

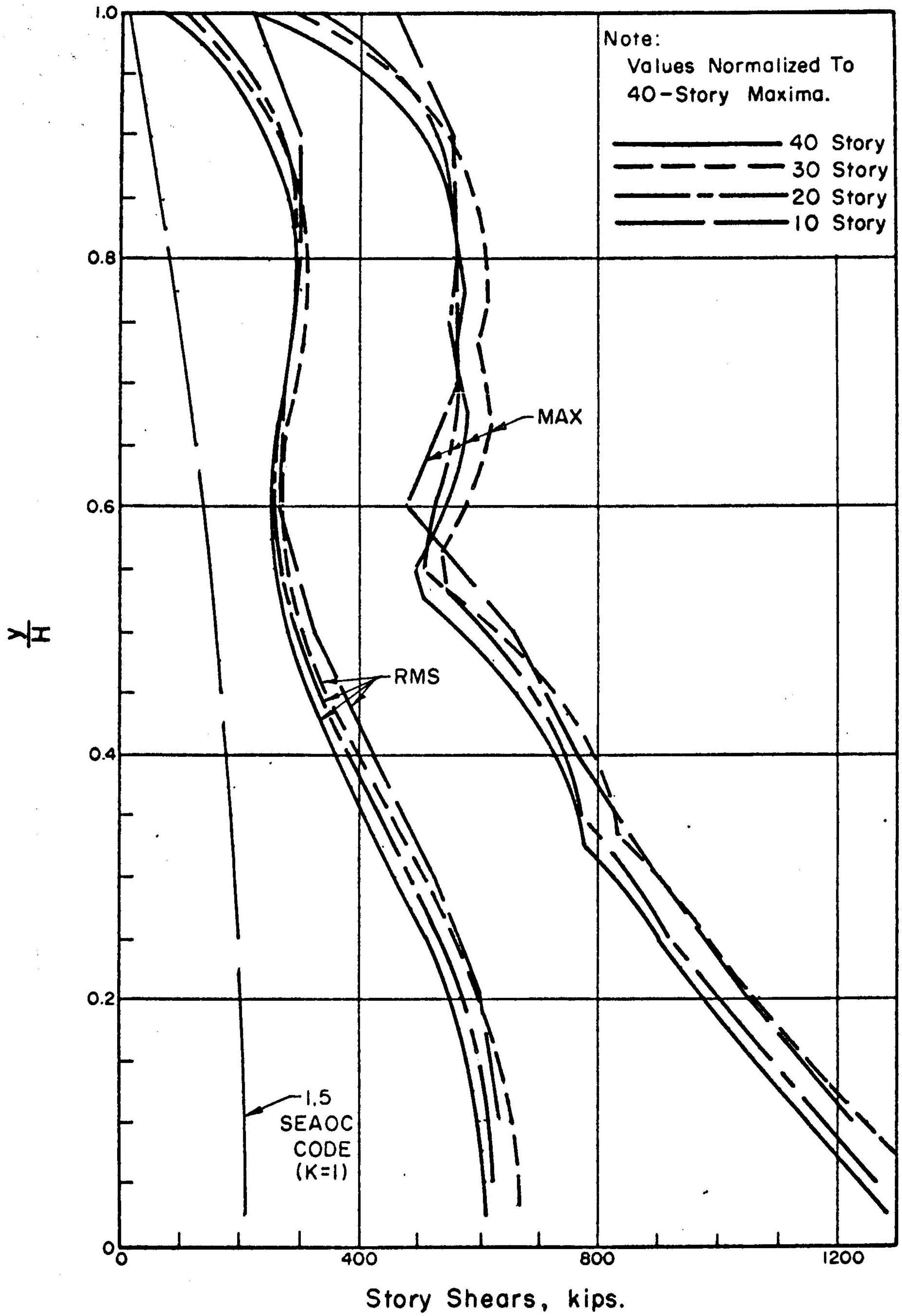


FIG. 18 MAXIMUM STORY SHEARS — 40-, 30-, 20-, AND 10-STORY FLEXURAL BUILDINGS, $T_0 = 3$ SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

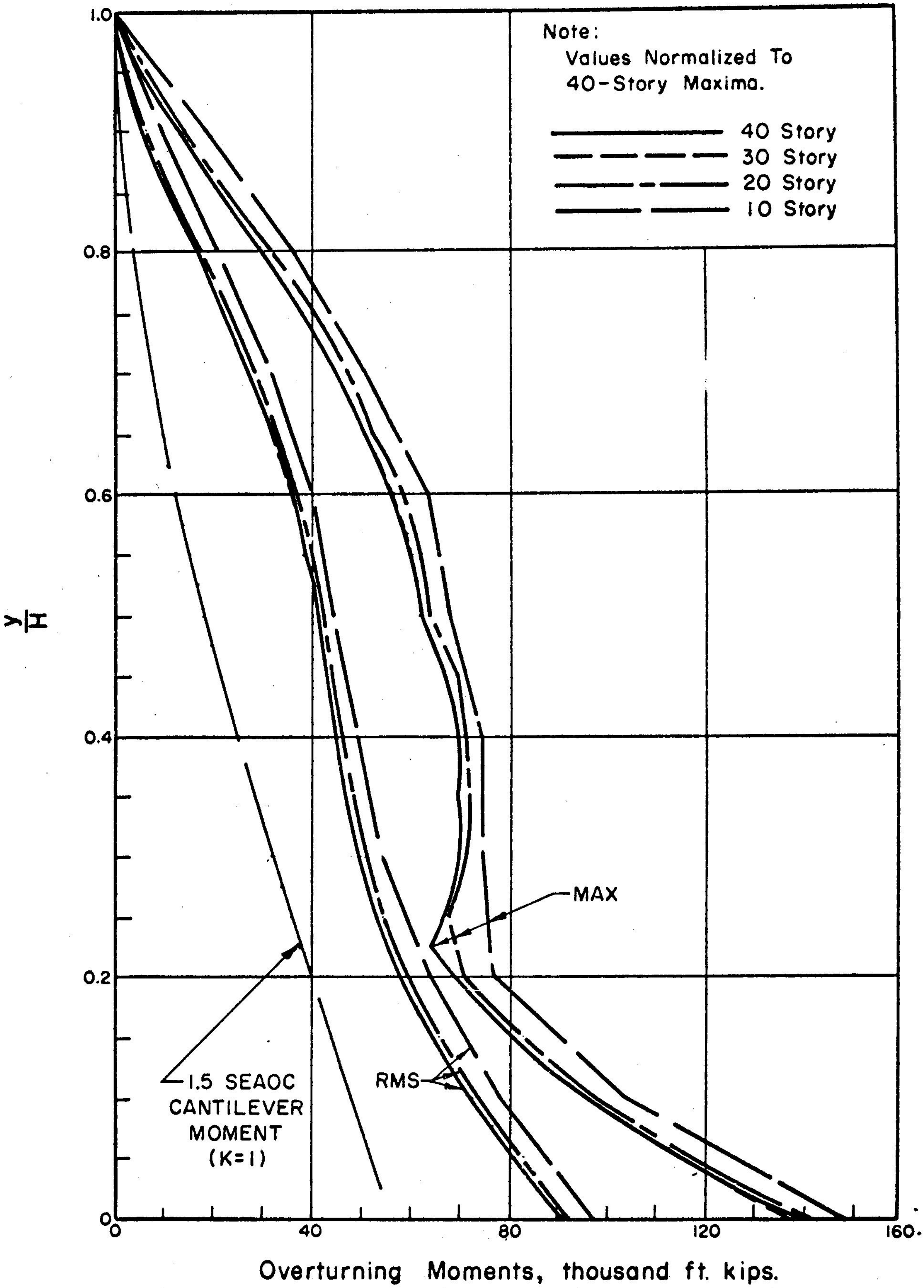


FIG. 19 MAXIMUM OVERTURNING MOMENTS — 40-, 30-, 20-, AND 10-STORY FLEXURAL BUILDINGS, $T_0 = 3$ SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

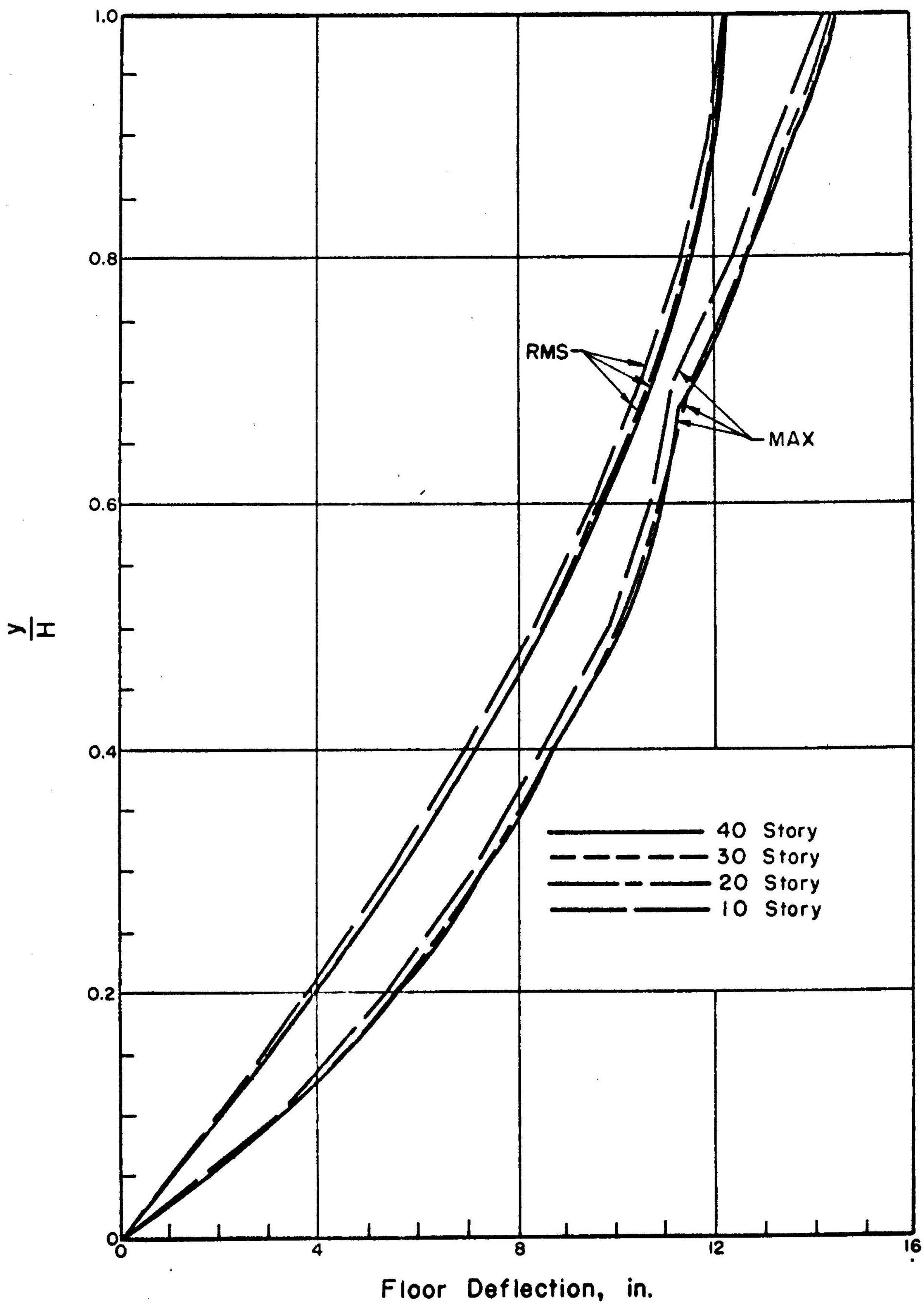


FIG. 20 MAXIMUM FLOOR DEFLECTIONS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 3 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

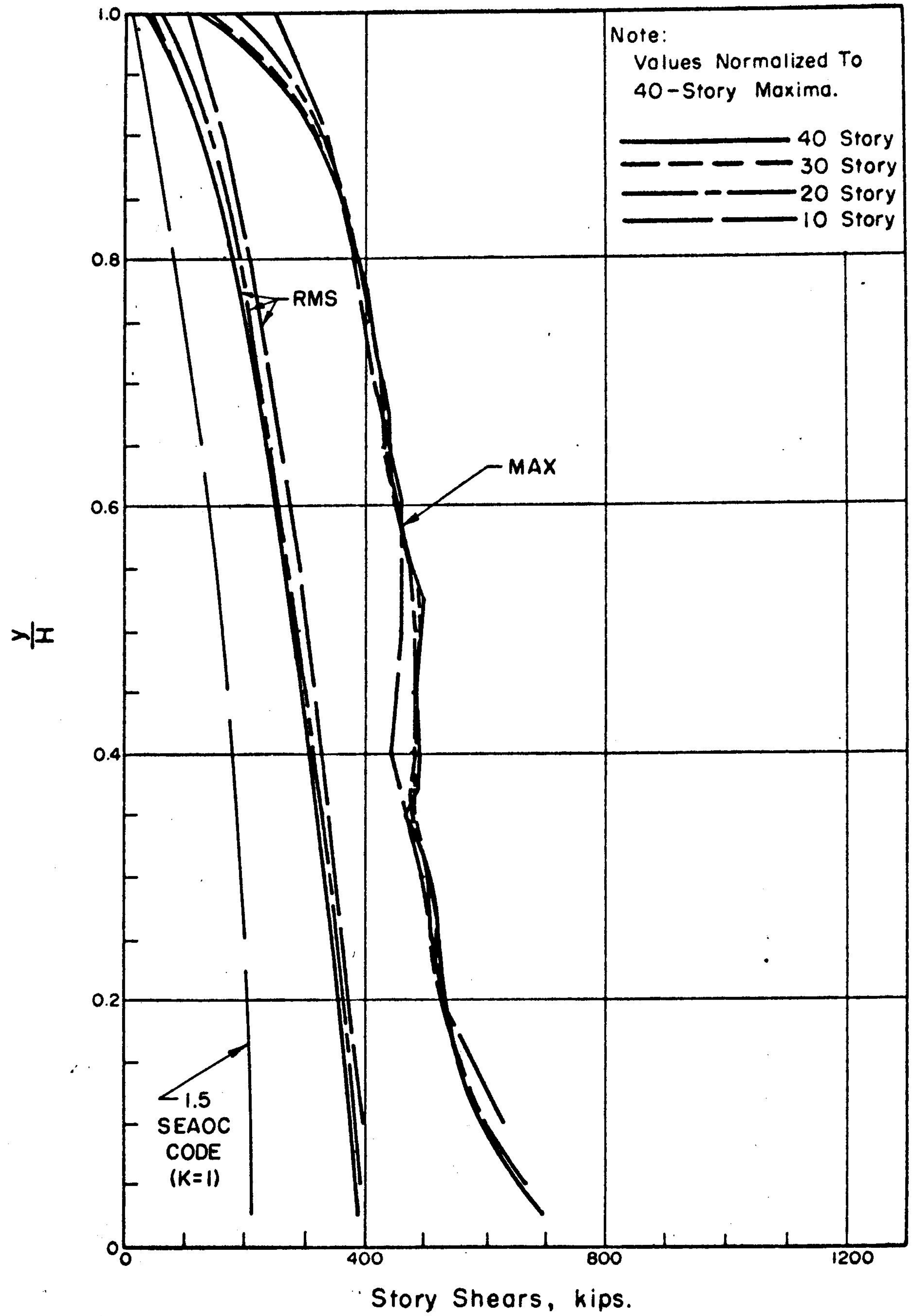


FIG. 21 MAXIMUM STORY SHEARS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 3 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

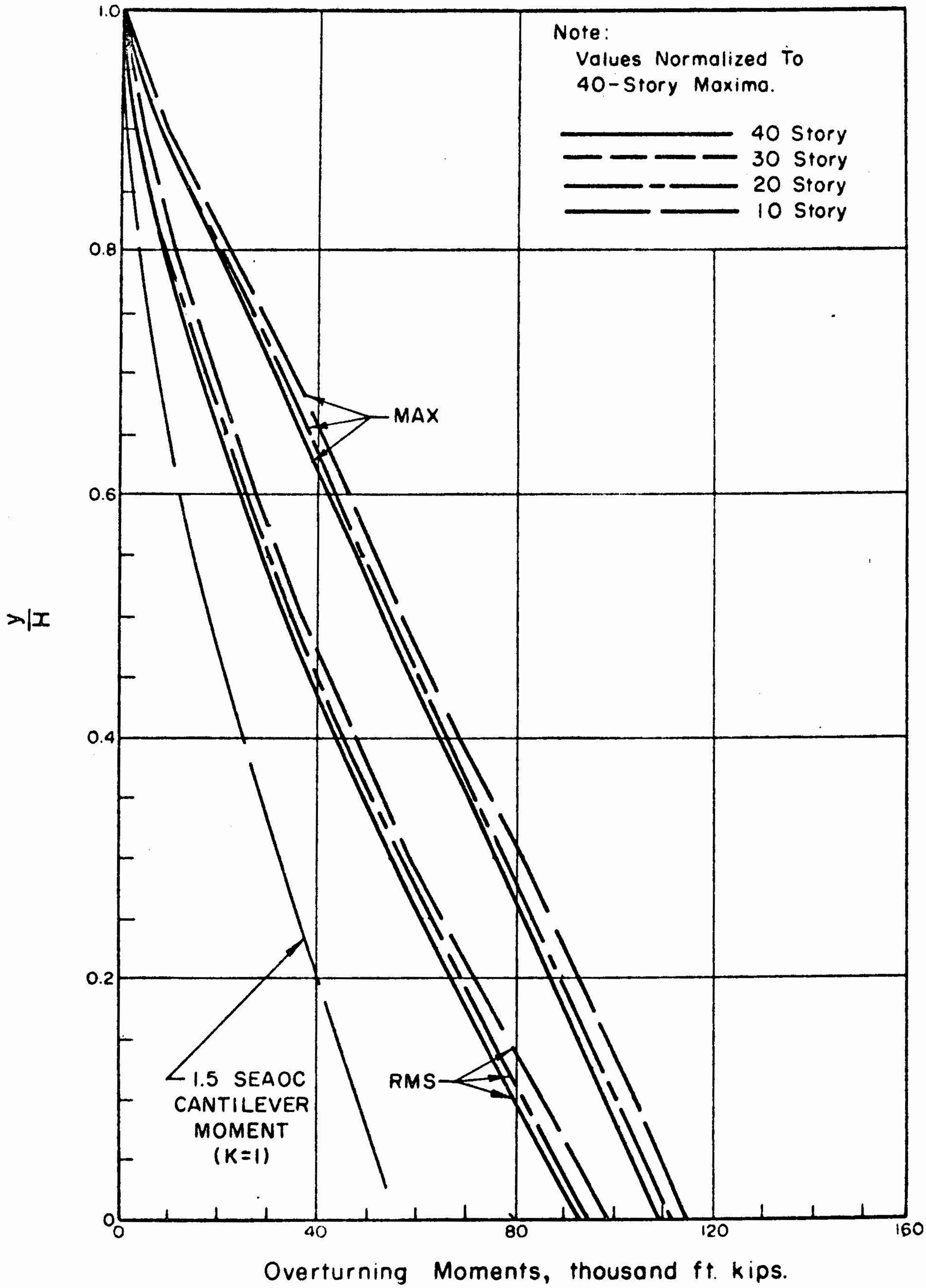


FIG. 22 MAXIMUM OVERTURNING MOMENTS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 3 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

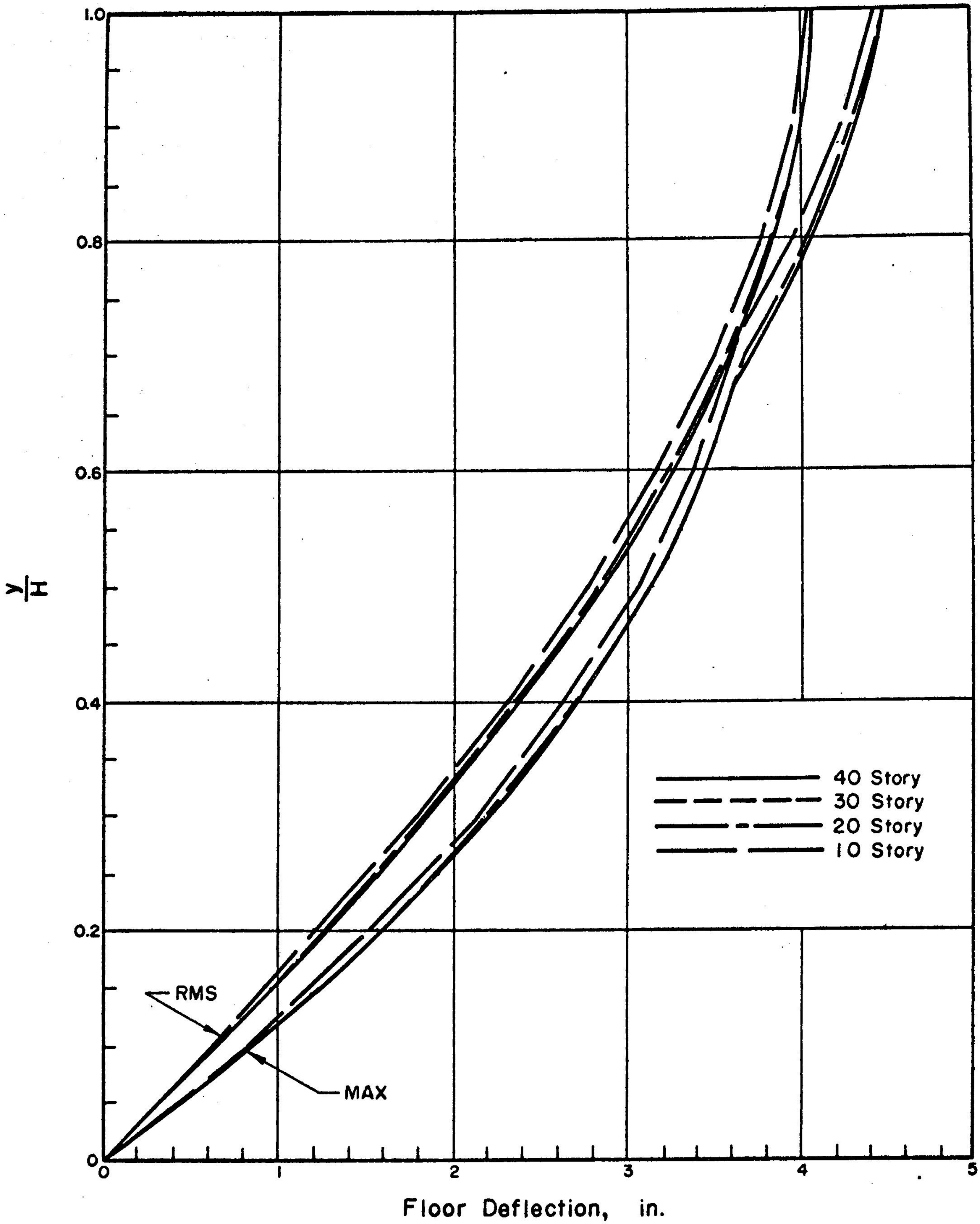


FIG. 23 MAXIMUM FLOOR DEFLECTIONS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 1 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

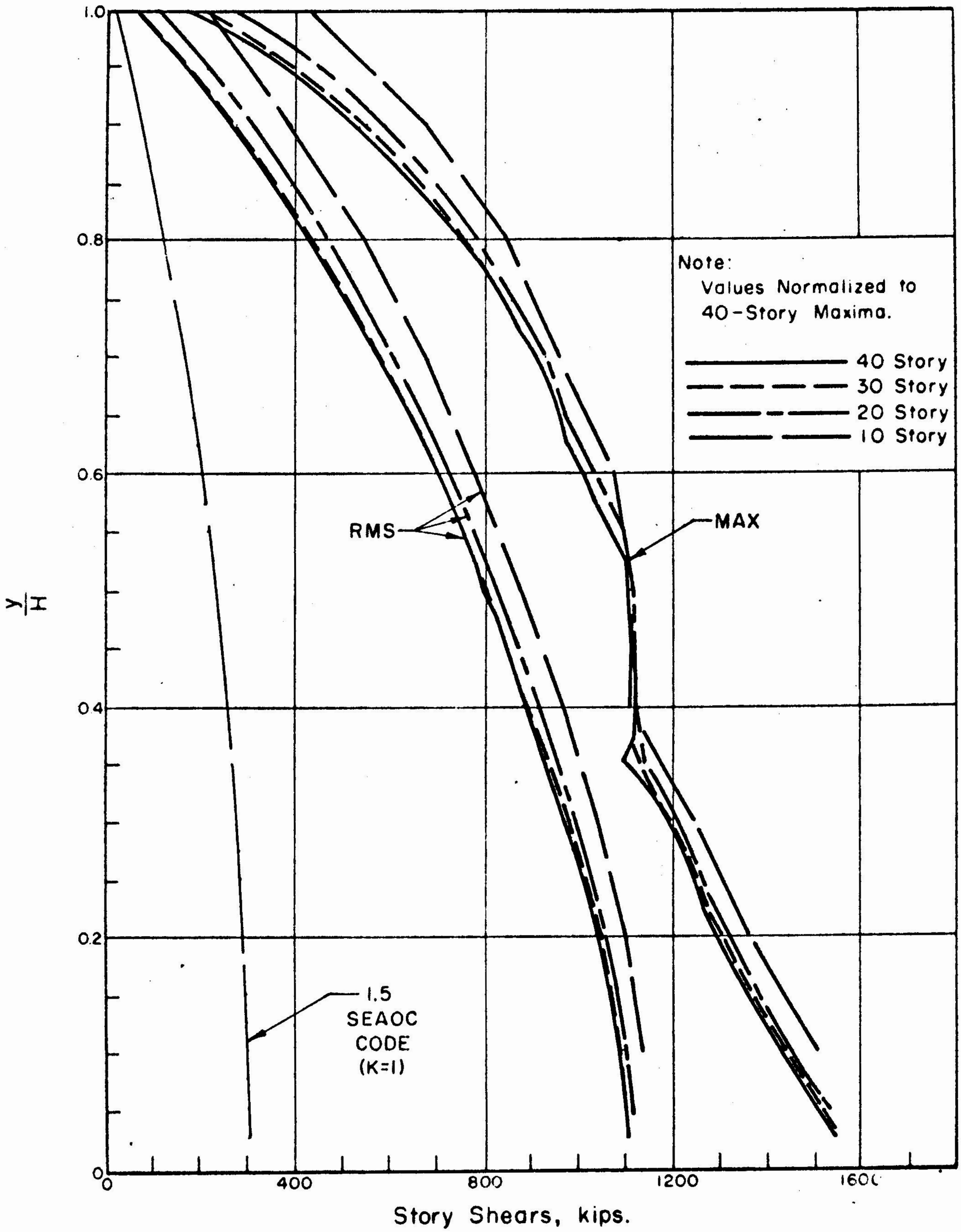


FIG. 24 MAXIMUM STORY SHEARS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 1 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

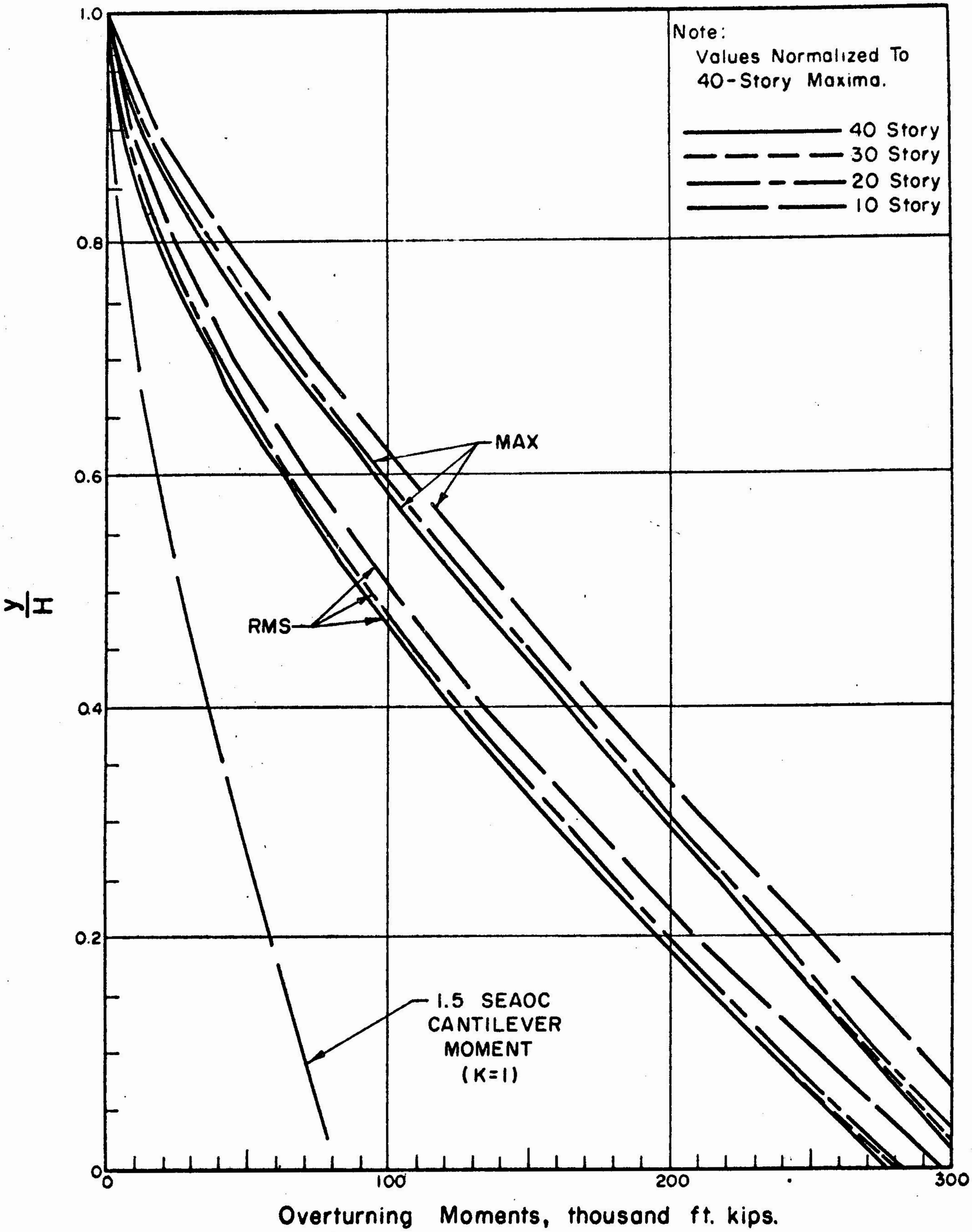


FIG. 25 MAXIMUM OVERTURNING MOMENTS — 40-, 30-, 20-, AND 10-STORY SHEAR BUILDINGS, T_0 = 1 SEC., D = 10 IN., V = 20 IN. PER SEC., A = 0.667 g.

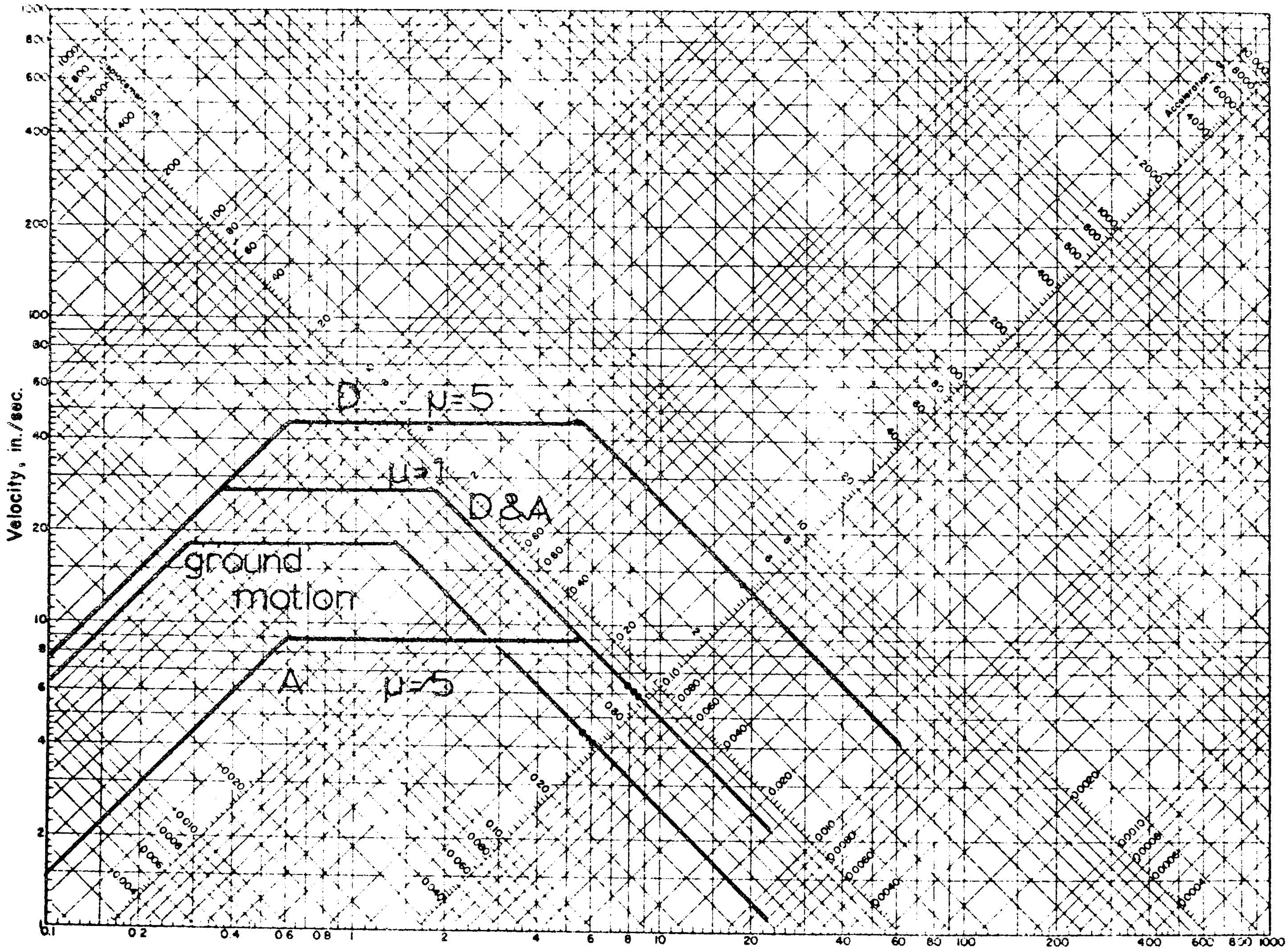


FIG. 26 COMPARISON OF ELASTIC AND ELASTO-PLASTIC SPECTRA

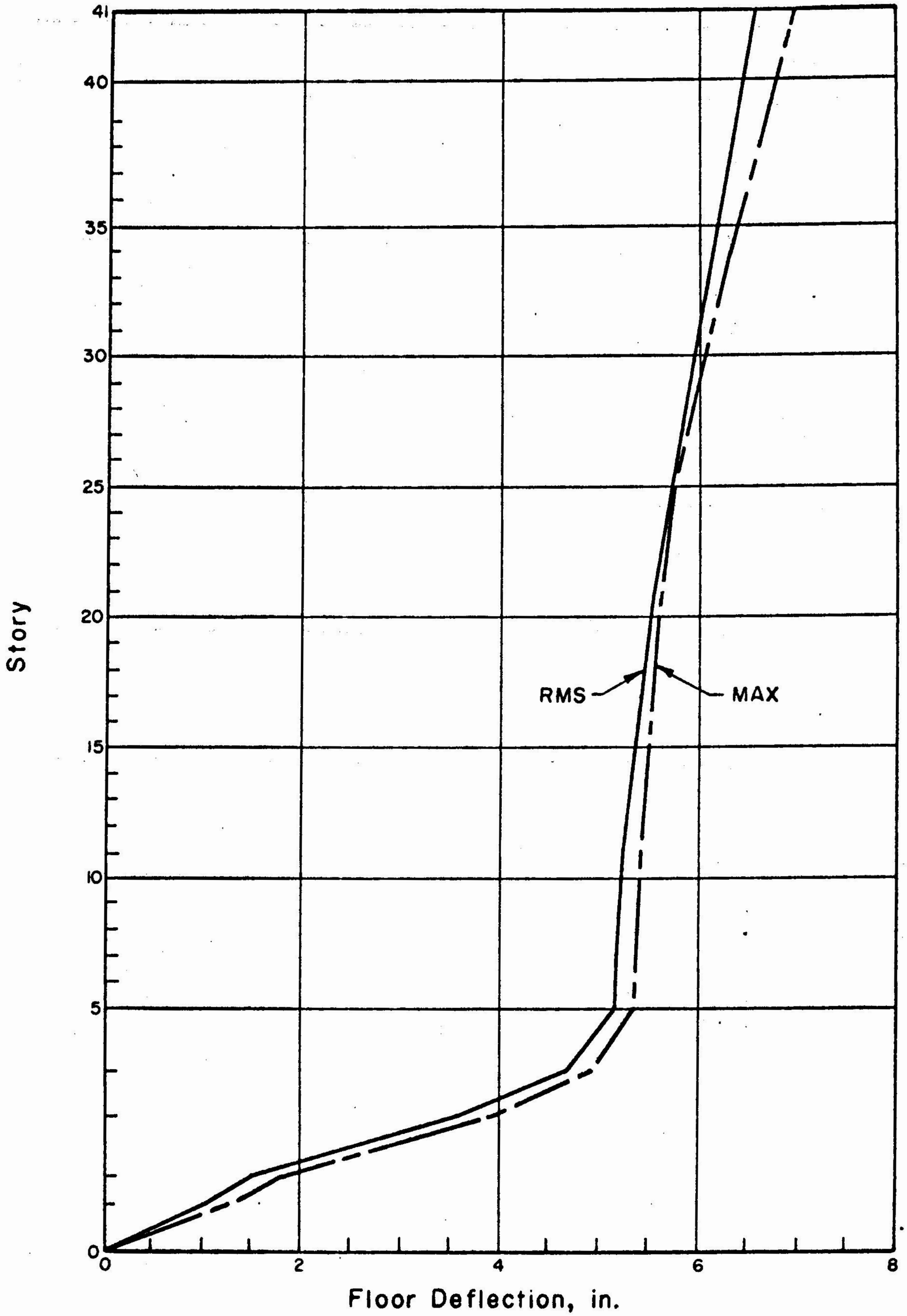


FIG. 27 MAXIMUM FLOOR DEFLECTIONS — COMPOSITE BUILDING, T_0 = 2.324 SEC., D = 6 IN., V = 15 IN. PER SEC., A = 0.5 g.

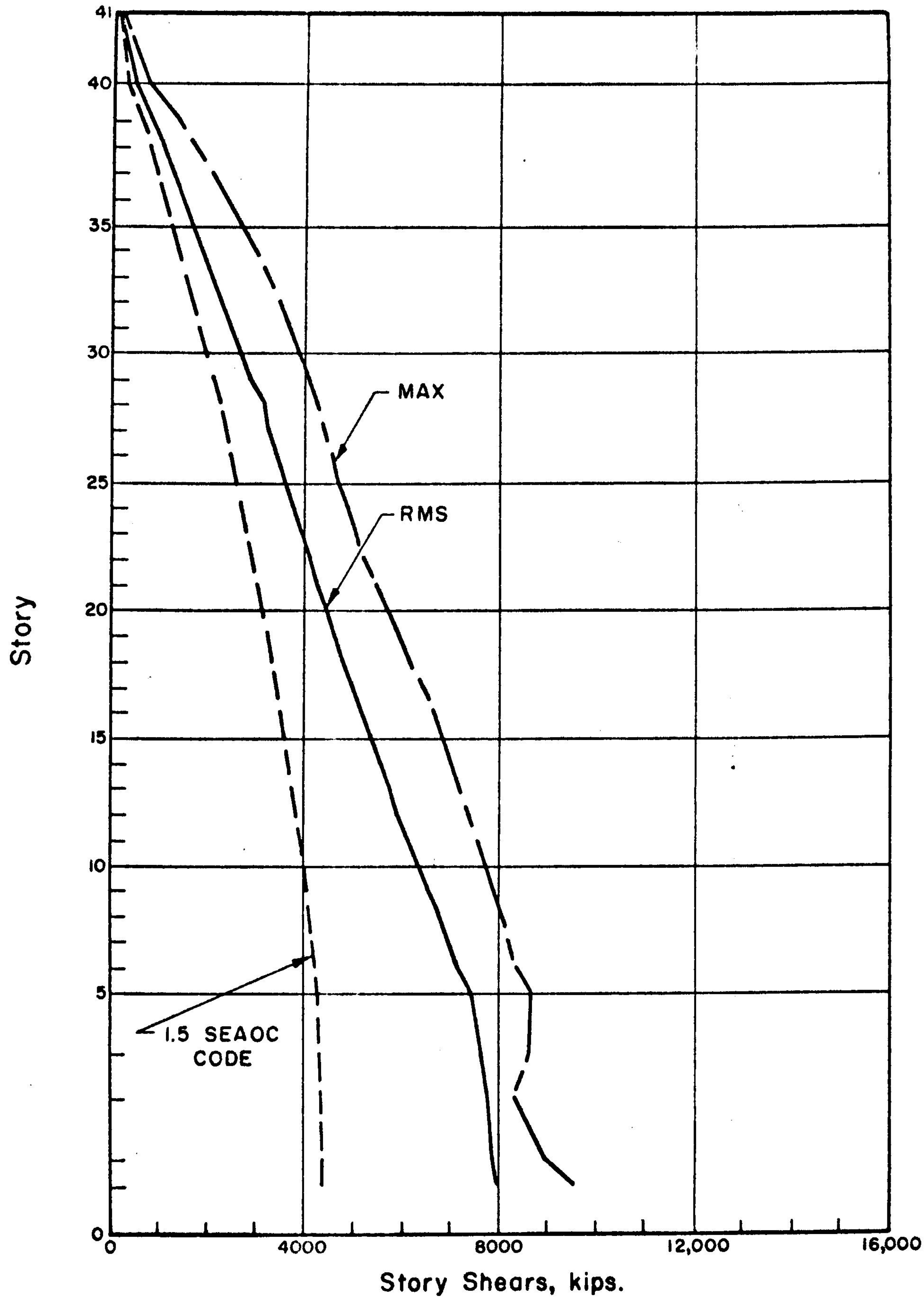


FIG. 28 MAXIMUM STORY SHEARS — COMPOSITE BUILDING, T_0 = 2.324 SEC., D = 6 IN., V = 15 IN. PER SEC., A = 0.5 g.

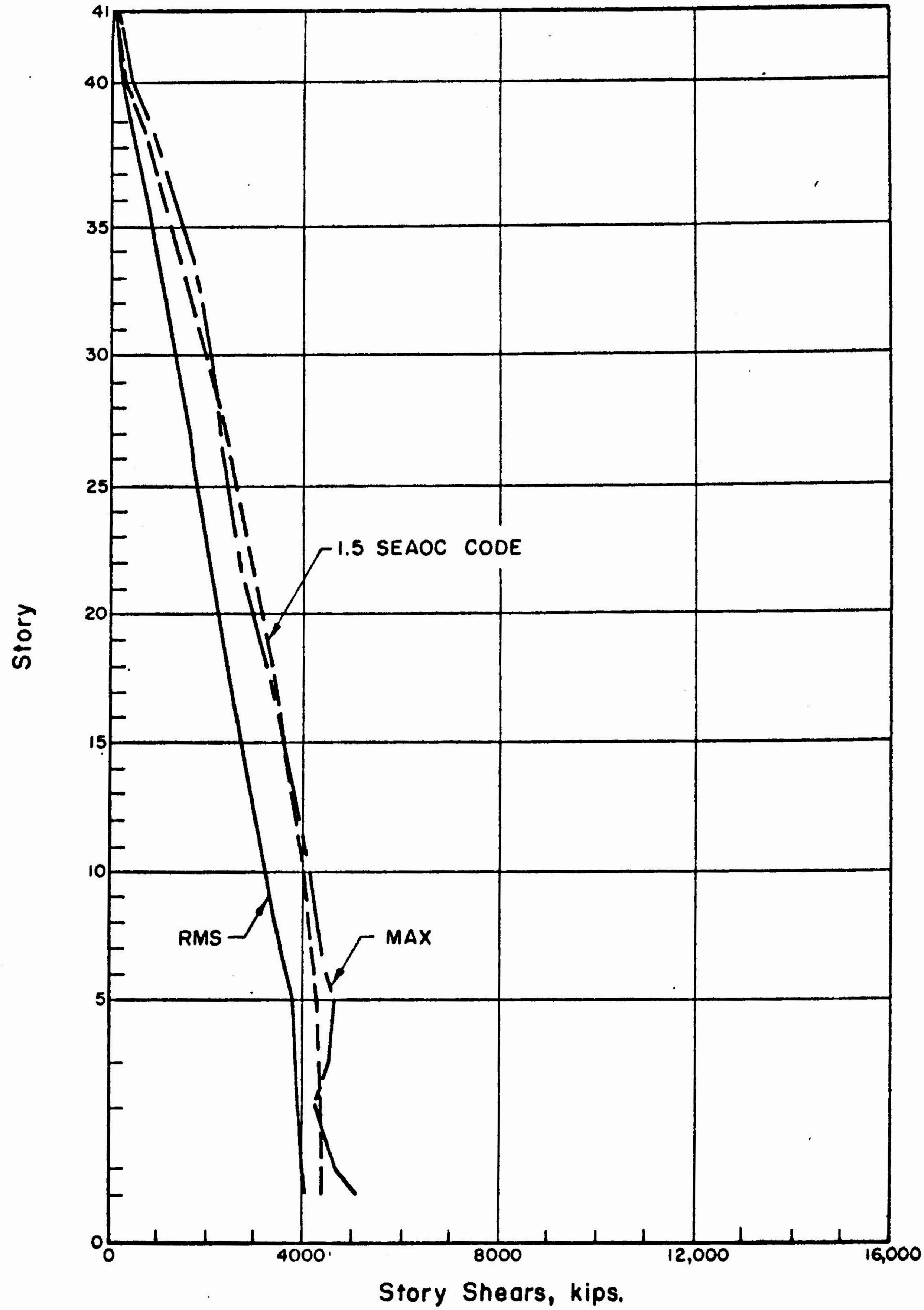


FIG. 29 MAXIMUM STORY SHEARS — COMPOSITE BUILDING, T_0 = 2.324 SEC., D = 3 IN., V = 7.5 IN. PER SEC., A = 0.375 g.

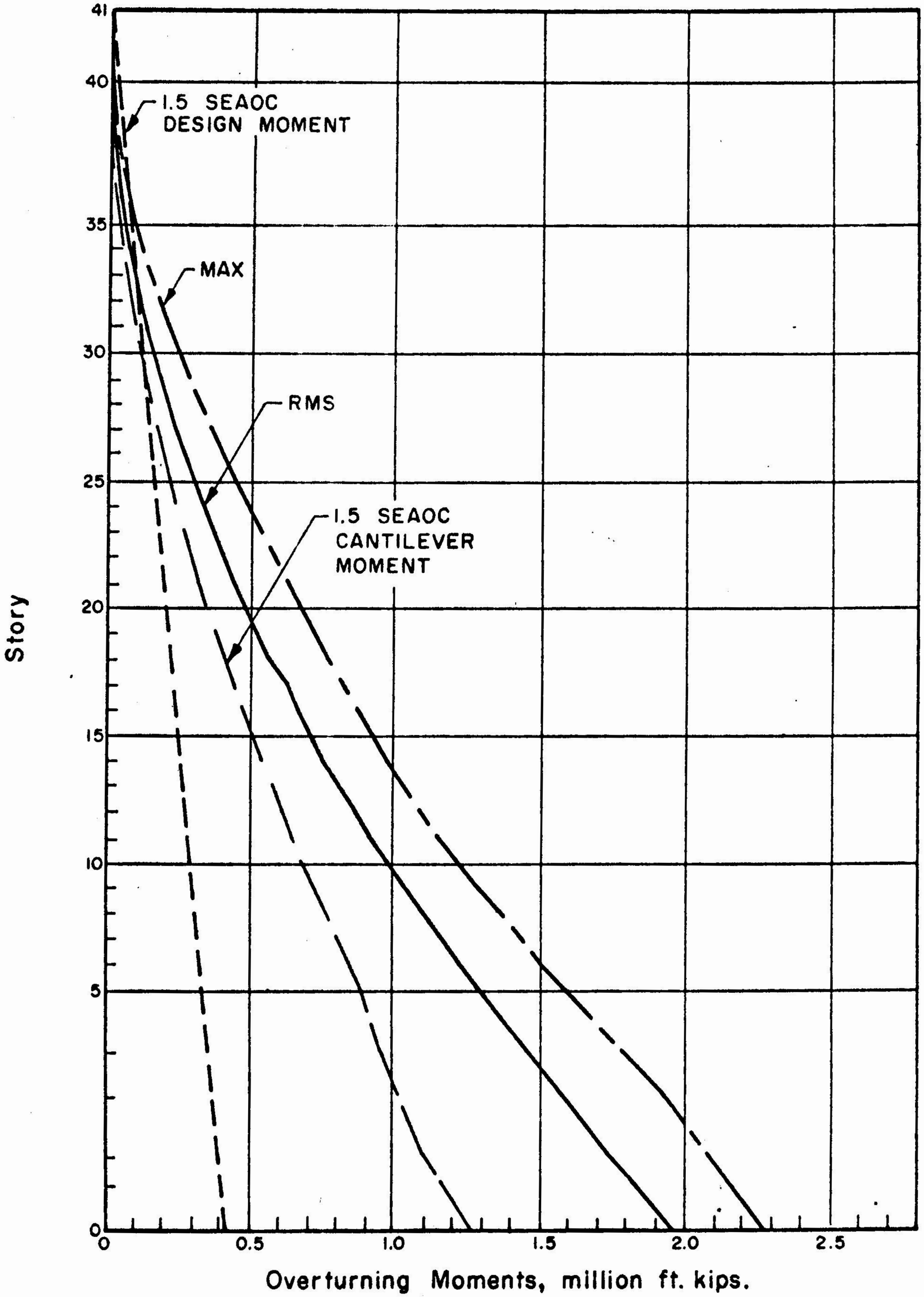


FIG. 30 MAXIMUM OVERTURNING MOMENTS — COMPOSITE BUILDING, T_0 = 2.324 SEC., D = 6 IN., V = 15 IN. PER SEC., A = 0.5 g.

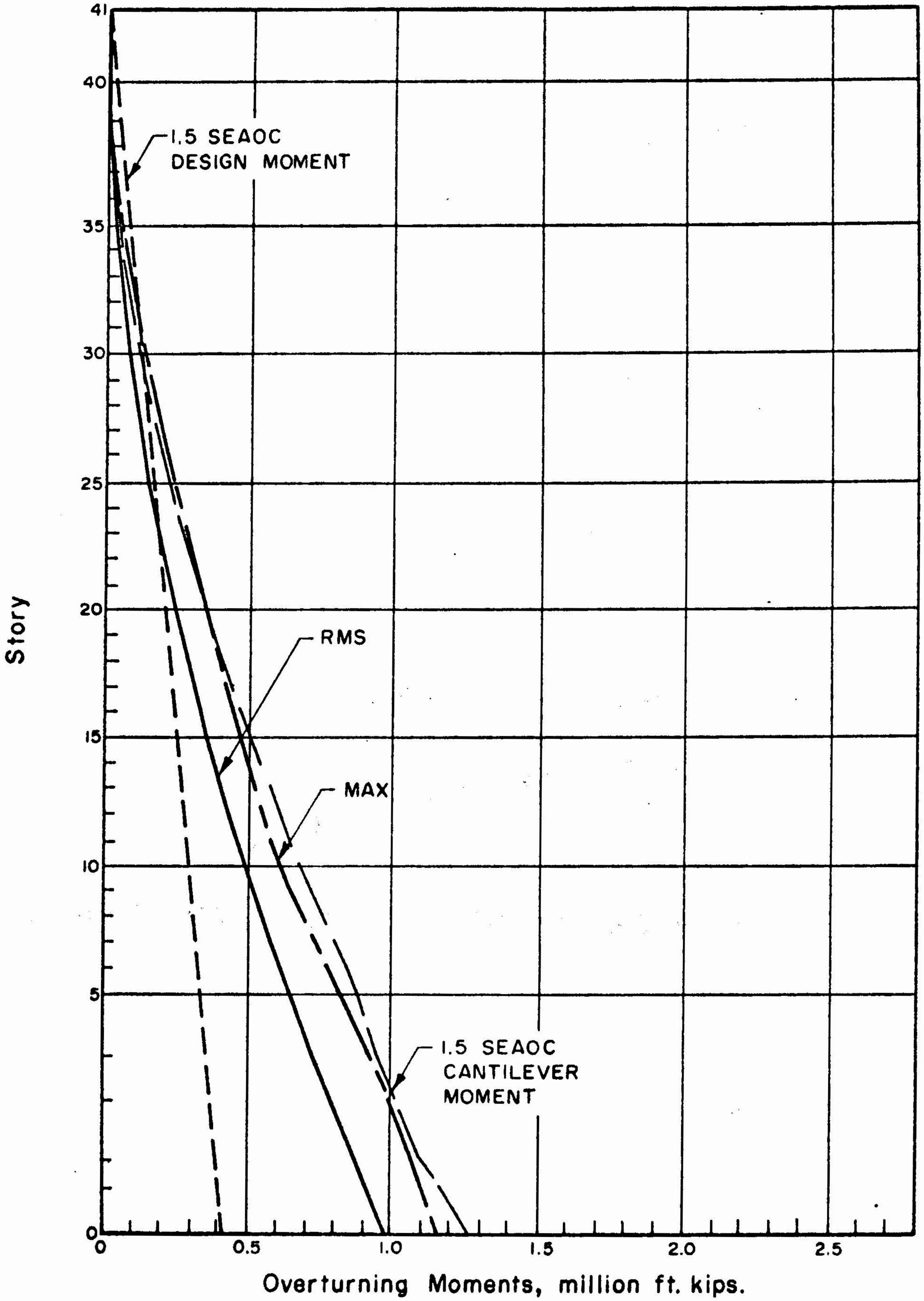


FIG. 31 MAXIMUM OVERTURNING MOMENTS — COMPOSITE BUILDING, T_0 = 2.324 SEC., D = 3 IN., V = 7.5 IN. PER SEC., A = 0.375 g.

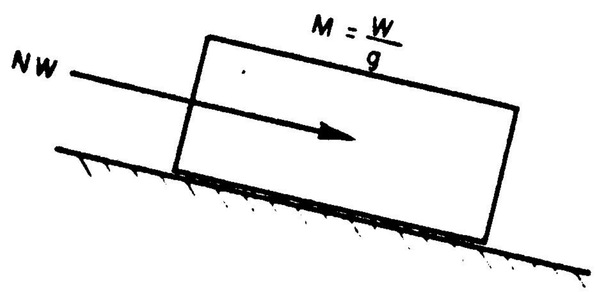


FIG. 32 MASS SLIDING UNDER CONSTANT FORCE

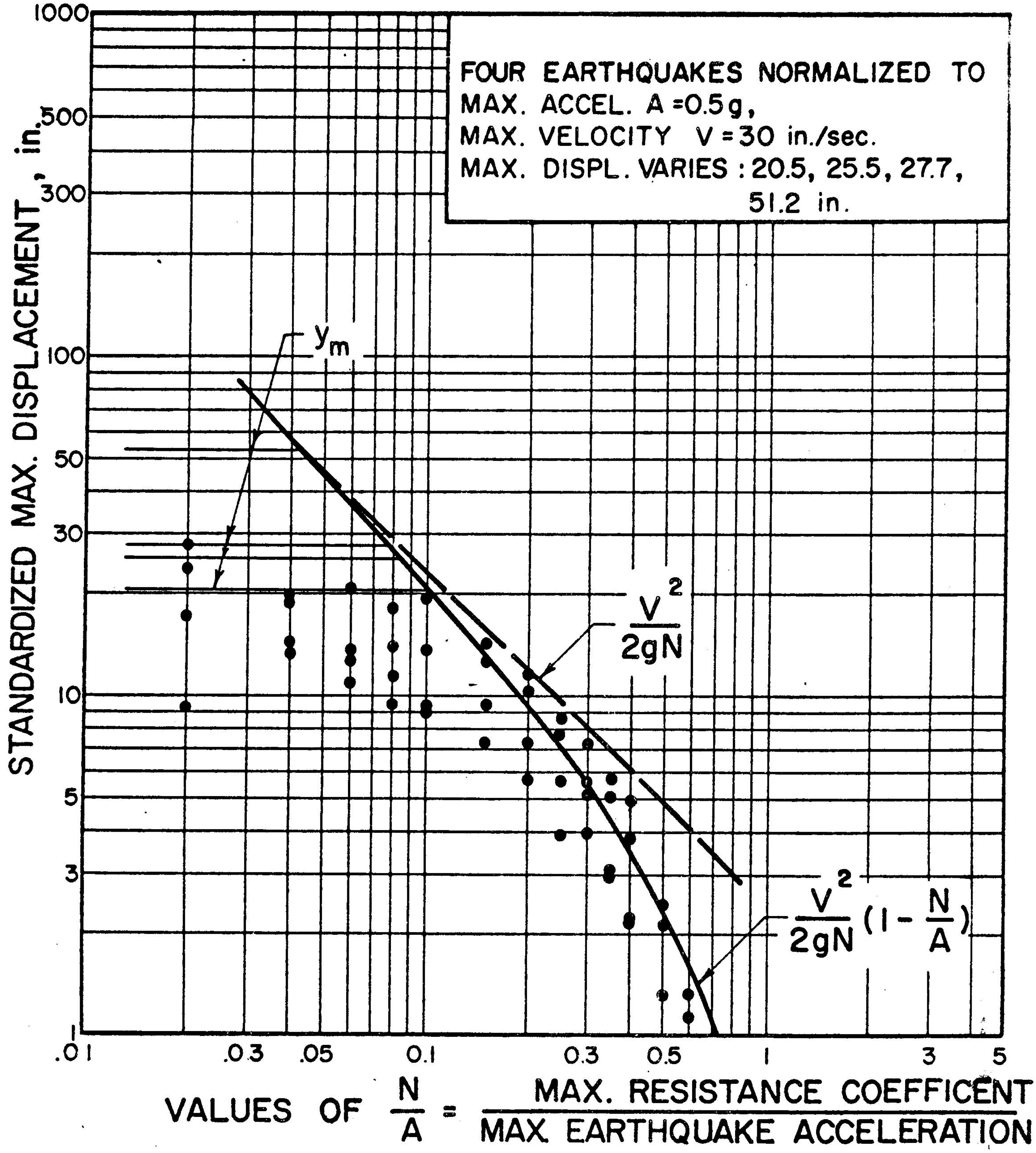


FIG. 33 STANDARDIZED DISPLACEMENT FOR NORMALIZED EARTHQUAKES. (SYMMETRICAL RESISTANCE)

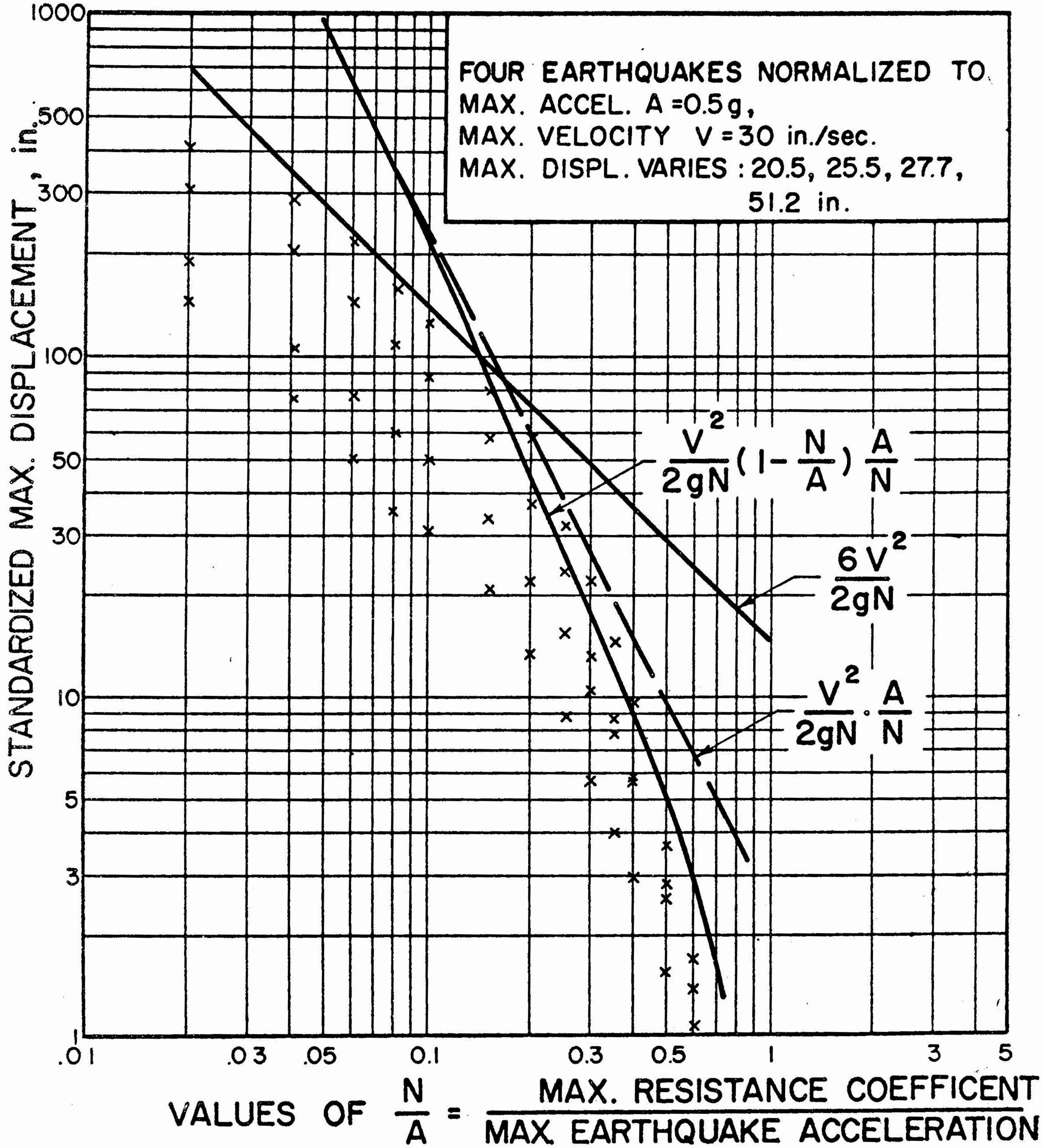


FIG. 34 STANDARDIZED DISPLACEMENT FOR NORMALIZED EARTHQUAKES. (UNSYMMETRICAL RESISTANCE)

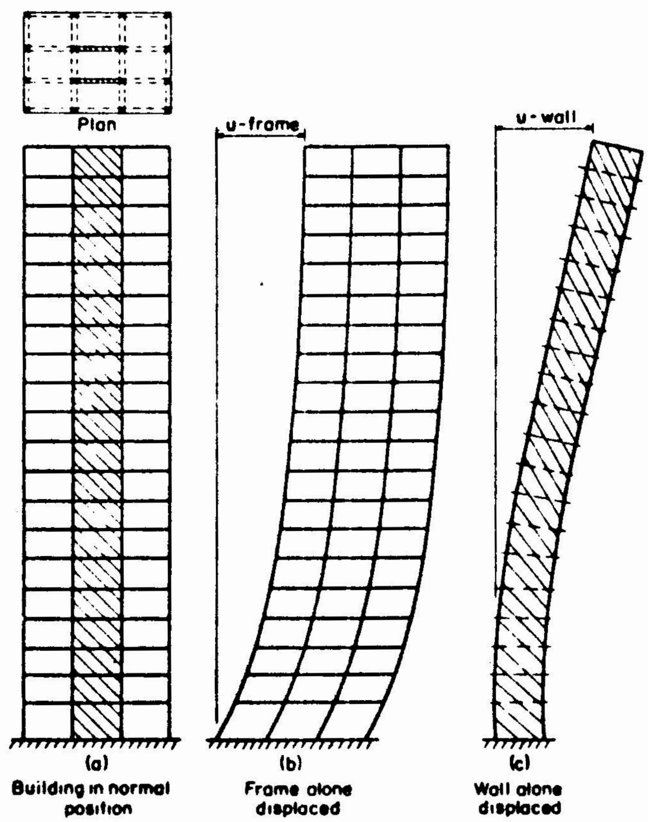


FIG. 35 TALL BUILDING WITH MOMENT-RESISTING FRAME AND SHEAR WALLS IN CENTER INTERIOR BAY